Microseisms Associated with Disturbed Weather in the Indian Seas

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VIII. *Microseisms Associated with Disturbed Weather in the Indian Seas.*

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(*Communicated by Sir* GILBERT T. WALKER, *F.R.S.*)

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[PLATE 1.]

CONTENTS.

1. *Introduction.*

Seismologists have often noted the appearance of pronounced microseisms in seismic records when the weather has been disturbed over a neighbouring sea. For instance, the late Dr. KLOTZ suggested a relationship between disturbed weather in the north Atlantic and the largest microseismic movements at Ottawa. Dr. HARRISON, writing in 'Nature,' November 1, 1924, in continuation of a note* by the present writer, pointed out that well-marked microseisms in the Omori charts at Calcutta invariably confirmed other evidence in the case of the early stages of dangerous cyclones, and were sometimes noticed when the storm centre was so much as 1000 miles south of Calcutta. He did not, however, recall any instance in which microseisms were associated with ordinary rough weather or with an advance of the monsoon. On the other hand, "investigation† at Eskdalemuir of the possible connection between microseismical amplitude and the state of the sea at different points of the British coasts have yielded results of an inconclusive kind. For example, the correlation between the state of the sea and the

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* 'Nature,' vol. 114, p. 576 (1924), vol. 116, p. 866 (1925), and vol. 123, p. 163 (1929).

† Meteorological Office Observatories, 'Geophysical Journal,' Annual Supplement, 1916, pp. 78–79. On pp. 119 and 120 of the 'Observatories' Year Book,' London, 1923, some relationship between the largest microseismic movements recorded at Eskdalemuir and Strasbourg and the disturbed weather in the neighbouring seas has been suggested.

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microseismical amplitude (N. component) at Eskdalemuir gives coefficients which are as high as 0·7 and 0·8 in winter, but which fall very low in summer."

In one of the publications* of the Zi-Ka-Wei Observatory, GHERZI has given interesting examples of microseisms recorded by Galitzin seismographs, which could be associated with cyclones, anti-cyclones and frost, and also some which were inexplicable. Following a suggestion made by G. W. WALKER,† GHERZI considers the microseisms to be due to sea waves caused by disturbed weather, but does not work out the consequences of this assumption in explaining the features involved in the different types of microseisms, which he associated with different types of weather.

GUTENBERG‡ has also given examples of microseisms of both short and long periods. He considers some of the short period microseisms to be due to breakers and the long period ones to be due to frost, but he, too, does not attempt to work out the physical consequences of the assumptions and then to explain the observed features in the different types of microseisms.

It is the object of the present paper to discuss the types of microseisms which are associated with disturbed weather in the Indian Seas, and to explain the observed periods and amplitudes by a mathematical analysis of the physical processes involved in the production of these movements.

Researches on microseisms have been based to a considerable extent on the records furnished by the Galitzin seismographs. This instrument, on account of its highly damped pendulum and a large adjustable magnification, is certainly most suitable for the study of the phenomena. The large magnification ordinarily employed in the Galitzin instrument has, however, its own disadvantage. The ground is never at rest, and even when the weather is apparently undisturbed over the neighbouring seas a Galitzin seismograph will show pronounced microseisms, and consequently those associated with a weather disturbance will be superposed on these ever-present movements. This makes it often extremely difficult to differentiate the type associated with any particular disturbance, and doubtless the complication caused by this superposition has retarded progress in the detailed study of the important features in the microseisms associated with the different weather types.

In 1923, I installed a Milne-Shaw seismograph, horizontal component (North-South), in an overground room of the Colaba Observatory. The seismograms were full of microseisms, which were sometimes strongly marked and sometimes feeble, but would never disappear from the records. They were very complicated movements and it was found extremely difficult to interpret them. It was therefore considered that the best arrangement for the study of microseisms and their relationship with weather

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* "Étude sur les Microséismes," 'Notes de Séismologie,' No. 5, Observatoire de Zi-Ka-Wei, 1924.

† 'Modern Seismology,' ch. 9, p. 74.

‡ 'Lehrbuch der Geophysik,' pp. 302–307. See also 'Seismische Bodenunruhe, Göttingen Dissertation,' 1911; "Bodenunruhe durch Brandung und durch Frost," 'Zeitschrift für Geophysik,' vol. 4, pp. 246–250, 1928; and 'Sonderabdruck aus Forschungen und Fortschritte,' December, 1928.

would be such an adjustment of the instrument so that it should just cease to record microseisms when the weather conditions were quiet over the neighbouring seas, such as we ordinarily get in the months of January and February, when the wind velocity seldom exceeds 20 miles per hour over the sea areas. If under such conditions microseisms make their appearance whenever the weather is disturbed either locally or over an appreciable part of the neighbouring seas, then their mutual relationship is clearly established. The various features observed in such microseisms can then be analysed and explained on theoretical grounds.

This ideal condition was secured not by altering the standard condition for the Milne-Shaw seismograph, namely, a period of 12 seconds for the pendulum, a damping ratio of 20 : 1 and an apparent magnification of 250 times, but by lowering the position of the instrument some 15 feet below the ground surface. The best working position for the instrument was found to be a solid masonry pillar of dimensions 6 feet by 4 feet by 2 feet 6 inches in an underground constant temperature room. The effect of this lowering below the ground surface was a considerable reduction in the amplitude of the microseisms and made the records just free from them on such days of the quiet months, January and February, when, judging from the published weather reports and the local observations, the weather over the neighbouring seas was undisturbed, and the wind velocity nowhere exceeded 20 miles per hour. When the instrument was working in the overground room, temperature changes and rainfall occasionally caused troublesome creep in the lines, making them sometimes highly congested. The installation of the instrument at a depth of 15 feet below the surface and the constant temperature in the room had the great advantage of keeping the lines in the record perfectly steady and equidistant even when large variations of temperature or heavy showers occurred outside.

The instrument has been maintained since 1924 in the condition referred to in the preceding paragraph by a frequent change of the boom point and examination of its efficiency, namely, the deflection in the light speck caused by giving the instrument a known tilt. Each day's record during the last five years was critically examined for microseisms. During the winter months, December to March, the records were found on some days to be remarkably free from microseisms, and on those dates when the microseisms did make their appearance in the records, no difficulty was experienced in associating them with some definite type of weather disturbance, mostly local and due to pronounced land and sea breezes. Those associated with local weather disturbance had invariably large periods, 10 to 30 seconds. They were quite pronounced when the wind velocity as recorded in the Observatory exceeded 20 miles per hour and invisible when it was 15 miles per hour or lower. Conversely, when the wind velocity exceeded 20 miles per hour they were looked for and were invariably found in the records.

Besides the above, two other definite types of microseisms were recognisable in the records and these were associated with—

(1) the south-west monsoon,

(2) the storms in the Arabian Sea and the Bay of Bengal.

Microseismic movements of a type, which were quite characteristic of the south-west monsoon period, made their first appearance in the seismograms generally in May with the advance of the monsoon in the south-east Arabian Sea, becoming more and more pronounced as the monsoon currents approached Bombay. They became less marked or disappeared during a temporary break in the monsoon and reappeared with the strengthening of the currents. They were more or less steady vibrations having periods ranging from 4 to 10 seconds.

Microseisms of the third type were by far the most interesting and were associated with storms in the Arabian Sea and the Bay of Bengal, and were very strongly pronounced when they caused rough seas over a fairly wide area. They have small periods, ordinarily 4 to 6 seconds, but their amplitudes show very characteristic irregular variations, suggesting superposition of waves of different periods. This feature is so conspicuous that the type is readily distinguished, and it becomes always possible to suggest the existence of storms as soon as such microseisms appear in the seismograms. It is well known that, in the Indian Seas, almost all the severe storms form during the pre-monsoon and the post-monsoon periods, and it is exactly during these periods that the records are almost free from monsoon microseisms. This makes it fairly easy to notice the gradual appearance of feeble microseisms of small periods but variable amplitude associated with the early stages of development of these dangerous storms. The microseisms become more and more marked as the storm becomes fully developed and comes nearer and nearer to Bombay, and disappear only after it passes inland and ceases to affect the sea. The intensity of the microseisms is thus found to be a function of the distance and the " severity " of the storm. During the few years the instrument has been in operation and perfect adjustment, a large number of storms formed in the Bay of Bengal and the Arabian Sea and all of them produced microseisms of the type referred to above. Conversely, this type of microseisms was not present in the records when there was no storm either in the Arabian Sea or the Bay of Bengal.

The identification of the microseisms associated with storms throws open to the meteorologists a new method of recording their existence. The analysis of the microseisms, elsewhere in this paper, associated with some typical storms will make it clear that, whenever the presence of this type of microseisms can be recognised in the record, they can be looked upon with confidence as indicating the existence of a storm. The information is thus a valuable aid to the forecaster, particularly when the indications regarding the existence of a storm from the usual meteorological sources are meagre.

The microseisms associated with the south-west monsoon have in all the years made their first appearance in the records long before it burst in Malabar. The incidence of a steady feeble vibration of periods 4 to 10 seconds (commonly of periods ranging from 4 to 7 seconds) is readily recognised in an otherwise undisturbed record, and thus affords a means of making a forecast of a temporary or the normal advance of the monsoon in the south-east Arabian Sea or the south Bay of Bengal long before its appearance in Malabar or the Burma coast.

Some typical microseisms of the different types have been reproduced in figs. 1 and 2. The periods and the amplitudes of the microseisms at the hours 0, 6, 12 and 18 are being tabulated for every day, beginning from the year 1924, and are being published in the Observatory Annual Volumes, to which reference is made for detailed information.

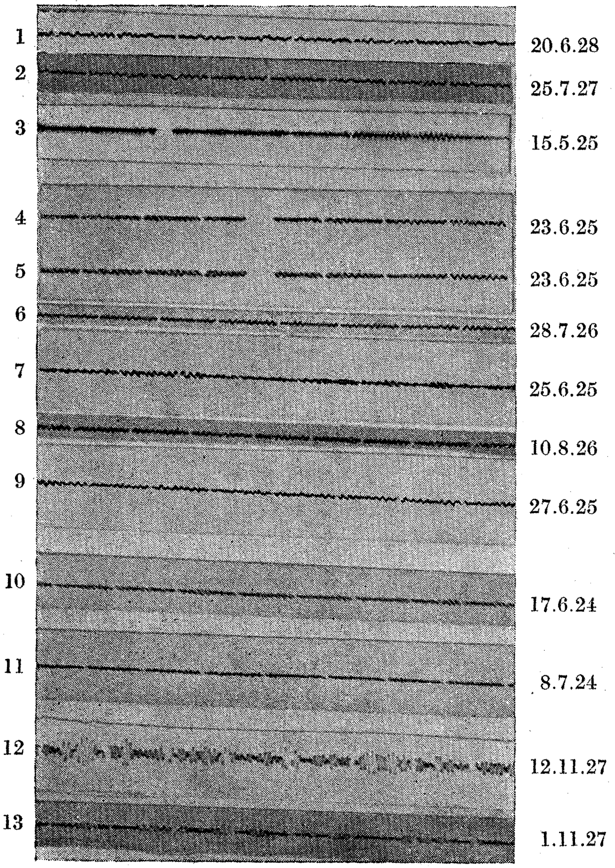

FIG. 1.—Short Period Microseisms associated with the S.W. Monsoon and Storms (N–S component). Instrument in the underground room. Interval between consecutive breaks equals one minute.

1, 2 and 4–11. Monsoon Type Microseisms.

3. Associated with a Storm in the Bay of Bengal.

4, 5, 7, 9. Associated with strong Monsoon with a depression in front moving into Kathiawar and causing rough seas off the Konkan and Kathiawar coasts.

12. Associated with a Storm in the Arabian Sea, which formed on November 11 and crossed coast between Bombay and Ratnagiri on 13th.

13. Associated with a Storm in the Bay of Bengal, which passed inland near Nellore on November 1.

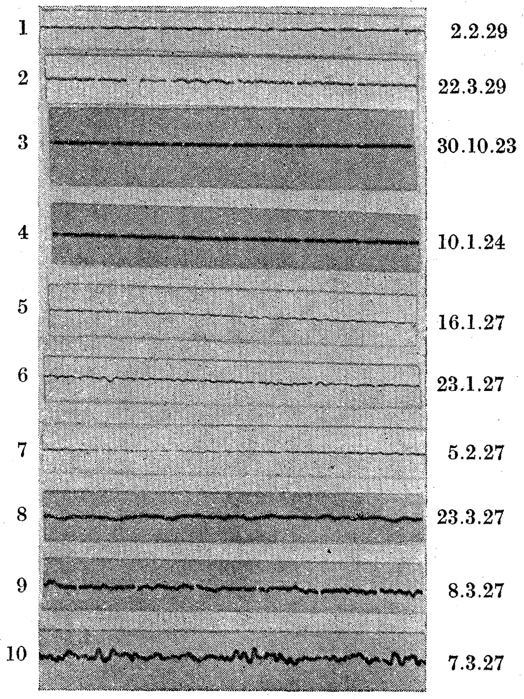

FIG. 2.—Long Period Microseisms associated with pronounced Land and Sea Breezes (N–S component). Interval between consecutive breaks equals one minute.

1, 7. Instrument in the underground room.

8–10. Instrument in an overground room.

2. *Microseisms Associated with the South-west Monsoon.*

Microseisms associated with the south-west monsoon are of the simplest type, and the manner in which they developed in the records in the different years with the incidence of the monsoon is very interesting.

For instance, in 1927, feeble microseisms of the monsoon type could be detected in the seismograms from the afternoon of April 29. They continued feeble on the 30th, but became fairly marked on May 1 and 2 and well-marked on the 3rd and 4th, showing at the same time slight variations in amplitude. The microseisms weakened on the 5th, but became well-marked again on the 6th and very strongly pronounced from the 7th to 10th. They became feeble again on the 11th and weakened still further on the 12th and 13th. The pronounced microseisms from the 1st to 11th were associated with a shallow depression which formed on the 2nd over the south-west of the Bay, developed into a storm on May 6, probably earlier, and crossed the coast near Akyab on the 8th. The microseisms became pronounced again on the 14th, strongly marked with variations in amplitude from the 15th to 18th and weakened on the 19th ; these were associated with a depression which formed in the Andaman Sea on May 14 and passed inland across the Arakan coast on the 18th. The microseisms became fairly marked on the 20th and 21st owing to an advance of the monsoon in the neighbourhood of Ceylon, well-marked on the 22nd, feeble again from the 23rd to 30th, strongly marked on May 31 and June 1, very strongly marked from the 2nd to 10th, slightly weakened from the 11th to 14th, but became very strongly marked again on the 15th and continued so till the end of July, with only slight variations in intensity, in almost perfect response to the strength and character of the monsoon. During this period the monsoon had established itself in Malabar on May 27 and at Bombay on June 11, and a depression had formed in the Arabian Sea west of the Laccadives on May 31 of whose movements very meagre information could be obtained at the time from meteorological sources. The strongly marked microseisms from May 31 to June 10, which showed variations in amplitude and were of the usual storm type, definitely suggest that the Laccadives depression did develop into a storm and exist in the Arabian Sea till the 10th, a fact which was subsequently confirmed by weather logs from ships. A break in the rains occurred early in August and the microseisms rapidly weakened. Owing to the weak character of the monsoon current that prevailed thereafter, they continued to be of feeble intensity in August and September, except when depressions formed over the head of the Bay of Bengal and affected the sea, and finally disappeared after the first week of October. Similar relationship between the strength of the monsoon current and the intensity of the microseisms has been found to be true in all the years the instrument has been in operation.

From the above account it will be clear that the microseisms of the monsoon type begin to make their first appearance long before the actual burst of the monsoon in Malabar, and that their development follows closely its progress and its subsequent history. If there is a strong monsoon over the sea areas the microseisms are strongly marked, and when the monsoon weakens, the microseisms, too, become weak.

It is, in fact, the roughness of the sea caused by the monsoon current that determines the intensity of the monsoon microseisms. To make this point clear, the amplitudes of the microseisms during the monsoon months of 1924 and 1925, and also the estimated state of the neighbouring seas, have been plotted in fig. 3. For the purpose of making this estimate, the figure 0 was assigned when the sea was smooth, 1 when it was slight, 2 when it was moderate, 3 when it was rough, and 4 when it was very rough at an observing station on any date, and the figures thus assigned to all the Indian and Ceylonese coastal stations were added together. The total figure was taken as a measure of the state of the sea. An estimate formed in this manner and based entirely on observations at coastal stations of the disturbance created in the sea by the monsoon current must be considered to be very rough, yet the curves in fig. 3 indicate a close parallelism between the state of the sea and the amplitude of the microseisms. The

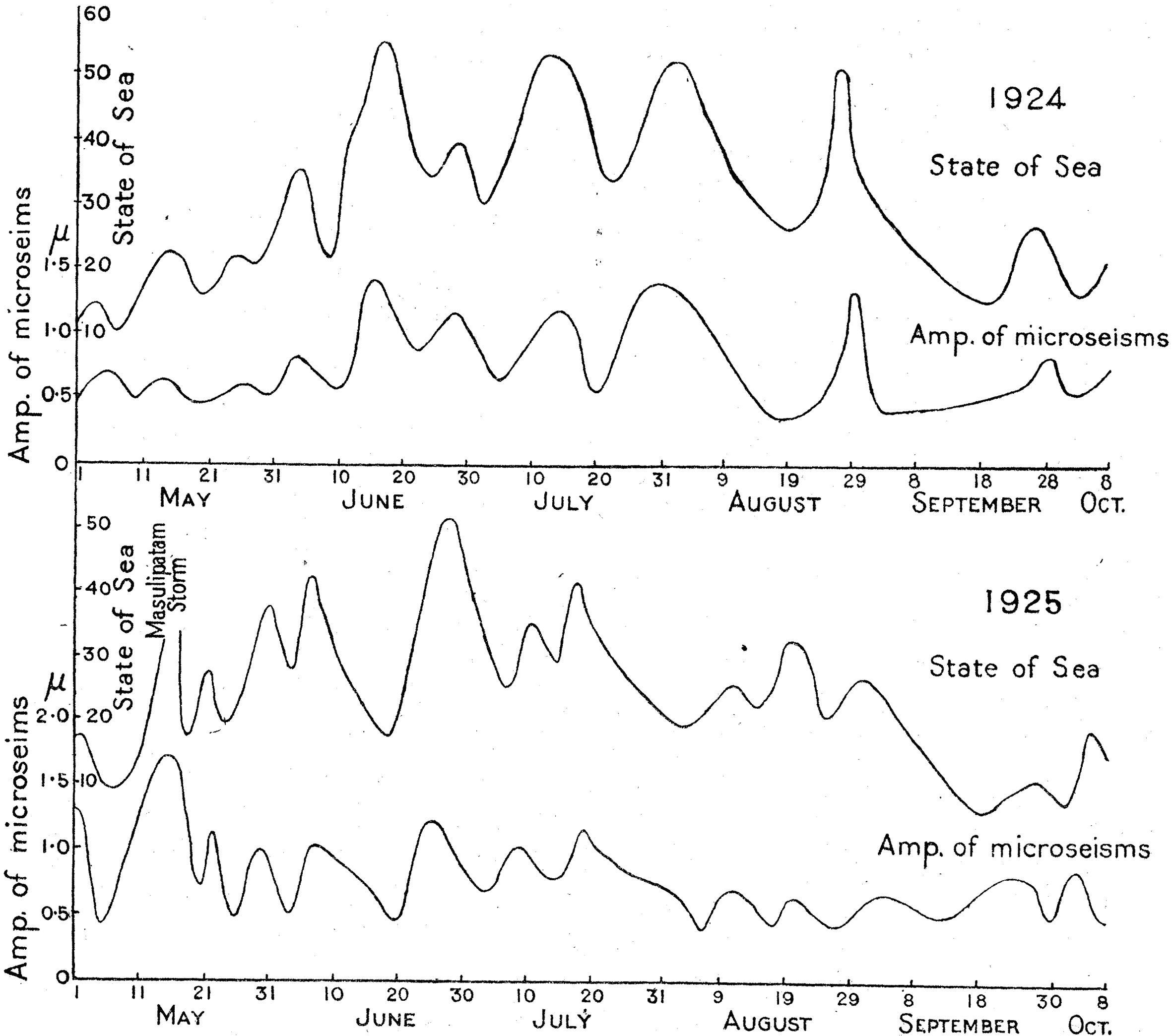

FIG. 3.—Relationship between Amplitude (N-S component) of Monsoon Microseisms and State of Sea.

agreement would undoubtedly have been still more close if a better measure could be obtained for the state of the sea, and also if the amplitude curve represented the amplitude of the resultant movements of the ground and not merely that of the north-south component.

The average monthly periods of the microseisms during 1924, 1925 and 1926 have been plotted in fig. 4. The curves show the manner in which the long-period microseisms give place to the short-period ones during the monsoon months.

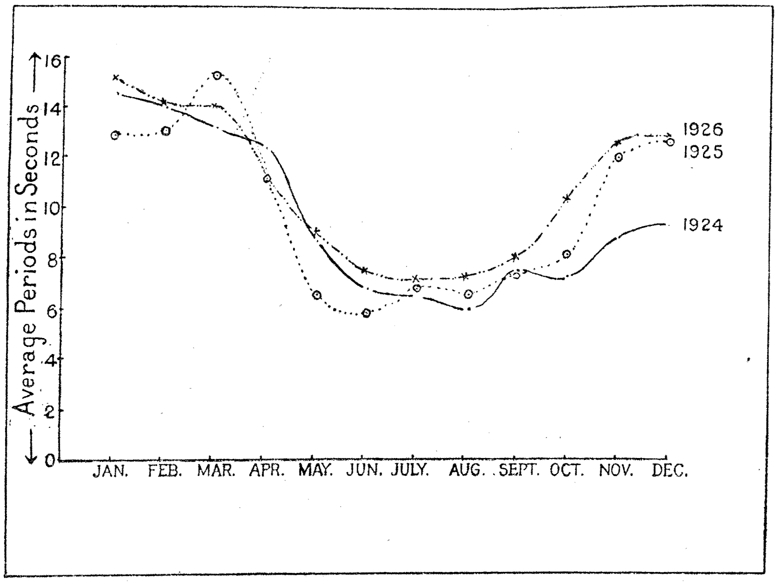

FIG. 4.

3. *Microseisms Associated with Storms in the Indian Seas.*

To illustrate the incidence, development, progress and final disappearance of microseisms produced by a tropical storm, the records for nearly 10 minutes from consecutive hourly traces obtained during a storm which formed in the Arabian Sea about 400 miles off the Konkan coast on November 11, 1927, and passed inland between Bombay and Ratnagiri on the 13th morning, have been reproduced in Plate 1. The most important peculiarity noticeable in these microseisms is the characteristic irregular variation in amplitude, a feature which enables them to be readily distinguished from those of the monsoon type reproduced in fig. 1. The manner in which the incidence of the microseisms occurred in the records is also worth noticing. The records show that before 18 hours on the 11th, there were no microseisms sufficiently pronounced to be recorded by the instrument. They began to make their appearance after that hour and rapidly developed from feeble movements to strongly marked vibrations. As the storm approached the coast, the microseisms became more and more pronounced. They weakened and finally disappeared after the storm passed inland on the 13th morning, slowly filled up, and then ceased to disturb the sea.

The appearance of this definite type of microseisms in an otherwise quiet record from the time of formation of a storm in the sea until its disappearance inland, noticed not only in the case of the storm referred to above but with all the storms which formed in the Arabian Sea and the Bay of Bengal during the few years the instrument has been in action, evidently prove that they could be due to no other agency but the storms.

The amplitudes of the microseisms are dependent on the distance and the intensity of the storms. The storms in the Bay of Bengal, on account of their distance from the Observatory, never gave such large microseisms as those reproduced in Plate 1. The type of the microseisms was, however, identical. For the sake of comparison, a cutting from the microseisms caused by the storm, which was known to have formed in the Bay of Bengal off the Coromandel coast, according to the meteorological information available at the time, on November 1, and to have crossed coast near Nellore on the 2nd, 10 days before the storm previously described passed inland, have been reproduced in fig. 1.

It is worth while noticing that although the type of microseisms produced by the two storms referred to above is identical, those associated with the Nellore storm do not show as much minor details as those associated with the Arabian Sea storm. This is due to the greater distance of the disturbed region from the Colaba Observatory in the former case. In the case of earthquake shocks, it is a common experience that the near shocks show much more complicated oscillations in all the three principal phases than the distant ones. On account of the longer distance of travel the minor details in the different phases are doubtless smoothed out by dissipation, and consequently the resultant vibrations are much more simpler than those produced by a near shock. The same cause undoubtedly contributes to make the microseisms produced by a distant storm much smoother than those produced by a near one.

In fig. 5 the average amplitudes of the microseisms (north-south component), recorded

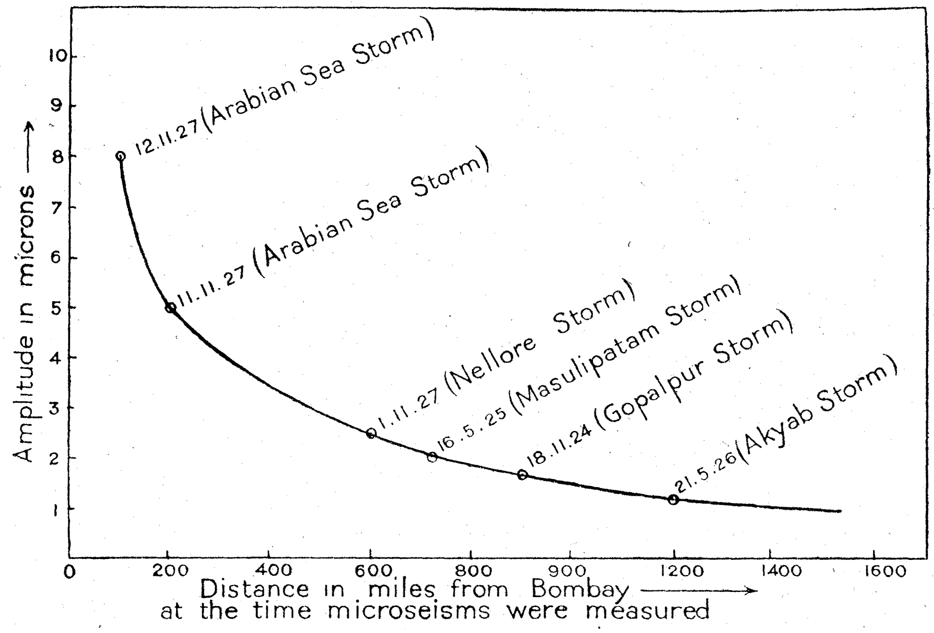

FIG. 5.

in the Colaba Observatory, have been plotted against the estimated distance from meteorological evidence, at the time at which the microseisms were measured, of some of the principal storms, for which good records are available. Assuming that the extent and the nature of the effect on the sea, produced by these storms, which were all severe, were identical, the curve indicates in a general manner the nature of the variation of the amplitude with the distance of a storm. It shows that the amplitude diminishes rather rapidly with distance for the first 600 miles and then very slowly with larger distances.

The variation of the amplitudes of the microseisms with the movements of an individual storm follows also somewhat the same law as will appear from fig. 6 in which the variations for two typical storms, one in the Arabian Sea and the other in the Bay of Bengal, have been plotted against time as well as their distance from Bombay. These curves show, as is to be expected, an increase in amplitude with the approach of the storm towards the observing station and then a decrease after the storm has crossed the coast, weakened into a depression, slowly filled up, and finally ceased to disturb the sea.

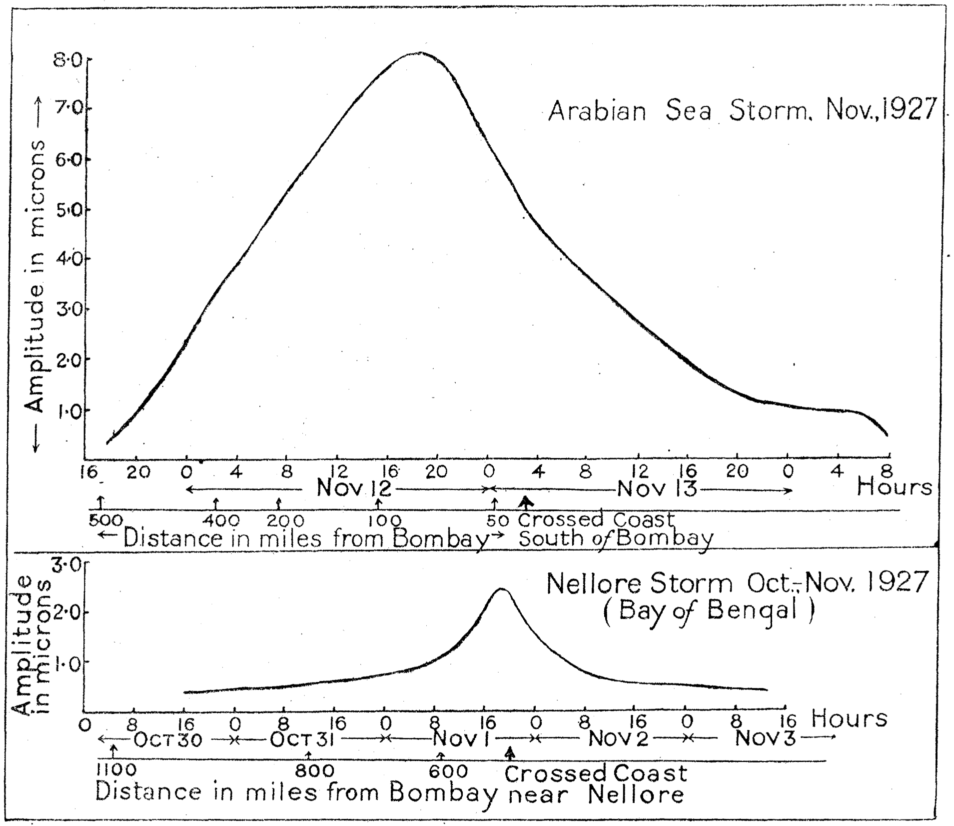

Fig. 6.

The wave systems produced by a storm in the Bay of Bengal have to travel through the Indian Peninsula, a strip of land about 550 miles in breadth, before they are recorded in the Colaba Observatory, and it is, therefore, remarkable that some of them cause ground movements in the Observatory as large as 2 μ. It would appear that the dissipation of energy during the travel of these microseisms is very small. One is thus led to consider that the microseisms are surface waves, penetrating, like the Rayleigh waves, very little into the interior.

The characteristic irregular variations in amplitude seen in the microseisms produced by storms are also shown during an advance of strong monsoon with a depression in front. For instance, the largest microseismic movements recorded during the monsoon months of 1924 were those of June 16 and the following days, and were associated with a low pressure area, with very strong monsoon to the south of it, which moved into Kathiawar and caused very strong winds off the Konkan and rough seas at Bombay. These, like those of the storm type, showed the usual irregular variations in amplitude. So also, in 1925, the largest microseismic movements recorded during the monsoon months were those associated with the strong monsoon which prevailed between June 22 and 30 to the south of a depression which moved into Kathiawar, and these had precisely the same characteristics. The irregular variations in the amplitude of the microseisms are due to the existence of a marked difference in wind velocity in the low pressure area and the surrounding regions as in the case of the storms. This will be made more clear when we enter into the details of the theory of this type of microseisms.

4. *Microseisms as an aid in forecasting disturbed weather over the Indian Seas.*

Once we are able to distinguish the different types of microseisms associated with different weather disturbances, it becomes a comparatively easy matter to make a forecast in regard to their existence. For instance, it will be readily seen from Plate 1 that if the photographic record was removed and developed at about 22 hours on November 11, 1927, the typical storm-type microseisms already strongly pronounced would have immediately given a warning of the existence of a severe storm. The information available at the time from the meteorological sources did not give any such definite indication at the forecasting centre until 8 hours of the next morning, when a message on the existence of this severe storm in the Arabian Sea was broadcasted.

Then again, in regard to a severe storm in the Bay of Bengal, which crossed the coast near Nellore on November 2, 1927, feeble microseisms could be detected in the record for October 29, on which date the meteorological evidence merely suggested the existence of unsettled conditions to the south-west of the Andamans. The microseisms became fairly pronounced on the 30th, but even on that date no definite information could be obtained from the meteorological sources, and the weather report merely stated that the unsettled conditions had probably developed into a depression off the Coromandel coast. On October 31 the microseisms were strongly marked, and on this date the weather report was that a deep depression had formed off the Coromandel coast and that it would probably develop into a storm. Such pronounced microseisms in an otherwise quiet record produced by a weather disturbance situated such a long distance away from the Colaba Observatory leave no doubt that the depression was a definite storm* on October 30 and 31, and this is confirmed by weather logs subsequently collected from ships. The microseisms became still more strongly pronounced as the storm advanced towards the coast on November 1 and continued so on the 2nd, on which date it lay as a deep depression over the Madras Deccan and was still affecting the sea. The depression moved to Central India and then slowly filled up on the 5th morning, and the microseisms, too, disappeared from the record on that date.

We can refer here to the severe storm of November 7 to 18, 1926, in the Arabian Sea, of which practically nothing was known to the Meteorological Office at the time, and also for a long time afterwards until the ships' logs were collected. Although the seismograph was not in a very good adjustment at the time, still very feeble microseisms of the storm type could be detected in the records from the afternoon of the 5th. They continued to appear till the afternoon of the 17th, after which they disappeared.

In view of the importance of microseisms for forecasting purposes in India, an analysis was made of all the storms and depressions which formed in the Bay of Bengal and the Arabian Sea during the years 1924–27 as shown in Appendix A for one of the years. The information given from meteorological sources in this Appendix has been extracted from 'India Weather Review and Annual Summary,' and does not, therefore, represent what was known in regard to the storms when they actually formed and travelled over the sea, but what was known about them after all the available ships' logs were collected, analysed and studied. It is found that in every case complete indication regarding the existence of the storms in the sea can be obtained from the microseisms. The analysis shows that some of the storms, which are declared to be severe from meteorological evidence, did not produce very well-marked microseisms. This was apparently due to the fact that some storms are of very small diameter and affect only a small area of the sea. The energy communicated to the surface of the earth in such cases is small and the microseisms recorded at a distant Observatory are feeble. When recorded under the conditions adopted in the Colaba Observatory, the microseisms must be a valuable aid to the forecaster, particularly for storms in the Arabian Sea, where information from meteorological sources are often very meagre.

The method has been tried for forecasting purposes for nearly two years, and has been found to give generally indication of disturbed weather over the Indian Seas earlier than that obtained from the meteorological reports.

5. *Theory of Monsoon Microseisms.*

Microseisms present an interesting problem for solution. Although in recent years they have attracted considerable attention it does not appear that the theory of these

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* The term "depression" is limited to those cyclonic circulations in which the wind does not reach gale force. The depression becomes a storm when the wind in a part of the cyclonic area rises to gale force, *vide* 'Storm Tracks in the Bay of Bengal and Arabian Sea,' by Dr. C. W. B. NORMAND.

where p and ξ are connected by the relationship (3) and μ/σ has the appropriate value given in the last column of the above table.

It is not contended that the expressions (9) and (3) represent the actual nature of the surface waves over the Indian Seas during the monsoon months. They are, indeed, very complicated, owing to variations of wind over the different parts, its direction not being concurrent with the direction of waves and other factors. But, certainly, for the purpose of getting numerical estimates of the quantities which we want to verify and for calculating the pressure disturbance at the bottom of the sea, we can assume equations (9) and (3) to be a sufficient approximation to the true condition, particularly during a period when there is a steady wind blowing over the sea. The equations in any case give correctly the observed heights and the periods of the waves for different wind velocities.

7. *Elastic Waves Generated on the Earth's Surface by Sea Waves.*

We can now proceed to obtain mathematical expressions for the microseisms. The gravity waves generated and maintained by the monsoon current on the surface of the sea will produce a disturbance of pressure at its bottom, and this will give rise to forced elastic waves over its bed.

On the assumption that the motion is entirely irrotational, the velocity potential ϕ corresponding to the gravity waves represented by equations (9) and (3) is given by

$$\phi = \frac{0\cdot0025\sigma V^2 g}{4\mu\xi p^2} \cdot \frac{\cosh \xi (y + d)}{\cosh \xi d} \cos (\xi x - pt). \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Theory as well as observations suggest that the motion associated with the gravity waves is confined to a superficial layer.* Below this layer the motion depends entirely on the ocean current, which we shall neglect in so far as the present problem is concerned. The equation (10) can therefore at the best be considered to represent the motion very close to the surface for which equation (9) remains approximately true. Over deep water, that is to say, when d is many times larger than the wave-length,

$$\cosh \xi (y + d)/\cosh \xi d$$

is approximately equal to unity within the region in which equation (10) may be considered to remain valid, and it therefore reduces to

$$\phi = \frac{0\cdot0025\sigma V^2 g}{4\mu\xi p^2} \cos (\xi x - pt). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

It is important to realise that if equation (10) be considered to represent correctly the

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* JEFFREYS, "On Turbulence in the Ocean," *loc. cit.*

that the motion is approximately irrotational. Wind blowing over the roughened surface of the sea must be a series of travelling eddies, and these, besides producing surface waves, must also cause turbulence* in the sea mainly confined in its superficial layer. This assumption is not, therefore, strictly applicable to the motion under consideration. Assuming, however, as JEFFREYS has already pointed out,† that these equations are applicable as a first approximation to the problem of waves on water and that a steady air current is capable of giving rise to a tangential force of this kind, namely, a force which acts forwards on the crests and backwards on the troughs, changing sign at the nodes, we can proceed to obtain expressions for the waves maintained over the surface of the sea in terms of the velocity of the wind and then verify whether they will give values for the amplitude, period and wave-length for waves having different wave-velocities, agreeing with the average observed values of these quantities.

As a first step, we note, as JEFFREYS‡ has already explained, that the easiest waves for a wind to raise are always two-dimensional and are *gravity waves.* The periods and the wave-lengths of the waves generated are therefore connected by the well-known relationship

$$p^2 = g\xi \tanh \xi d, \quad \ldots \ldots \ldots \ldots \quad (3)$$

d being the depth of the sea. This equation has been discussed by AIRY.§ It is known that the numerical values furnished by this equation for the periods and wave-lengths of waves having different wave-velocities agree with the average observed values|| of these quantities. As we shall presently require these numerical values for a detailed study of the properties of these microseisms, we have collected them in Table I. Most of the figures in this table have already been published by AIRY and the others have been calculated.

To introduce the velocity of wind into our equations, we require a knowledge of the "skin friction." The determination of this quantity for surfaces having varying degrees of roughness has formed the subject of many experimental investigations. For the roughened surface of the sea, EKMAN¶ obtained from his theory of wind drift in an ideal boundless ocean the relation

$$\sin \gamma = \frac{3T}{2\rho g d}, \quad \ldots \ldots \ldots \ldots \quad (4)$$

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* JEFFREYS, 'Phil. Mag.,' vol. 39, p. 578 (1920).

† Regarding LAMB's method, JEFFREYS ('Roy. Soc. Proc.,' vol. 110, A, p. 241 (1926)) has remarked: "Being under the incorrect impression that LAMB's approximation would not hold for the short waves I was chiefly considering, I proceeded on more elaborate lines. It now appears, however, that LAMB's method is not only applicable to problem of waves on deep water but is readily extended to cover the case when the water is comparatively shallow and to allow for surface tension."

‡ 'Roy. Soc. Proc.,' A, vol. 110, pp. 244–245 (1926).

§ AIRY, 'Tides and Waves,' arts. 169, 170. Also LAMB's 'Hydrodynamics,' 4th edition, p. 357.

|| KRÜMMEL, "Handbuch der Ozeanographie," vol. 2, pp. 40–45.

¶ 'Arkiv. Mat. Ast. Fys.,' vol. 2, No. 11 (Stockholm, 1905), and 'Ann. Hydr.,' pp. 423 (1906), *et seq.*

TABLE I.—C = Velocity of waves in feet per second, T = Period in seconds.

Depth of water in feet.	Length of wave in feet.											
	10.		50.		100.		150.		200.		250.	
	C.	T.	C.	T.	C.	T.	C.	T.	C.	T.	C.	T.
10	7·1	1·4	14·7	3·4	16·8	6·0	—	—	—	—	—	—
50	7·1	1·4	16·0	3·1	22·5	4·4	—	—	—	—	—	—
100	7·1	1·4	16·0	3·1	22·6	4·4	—	—	—	—	—	—
1,000	7·1	1·4	16·0	3·1	22·6	4·4	27·6	5·4	31·9	6·3	35·7	7·0
10,000	7·1	1·4	16·0	3·1	22·6	4·4	27·6	5·4	31·9	6·3	35·7	7·0
20,000	7·1	1·4	16·0	3·1	22·6	4·4	27·6	5·4	31·9	6·3	35·7	7·0

Depth of water in feet.	Length of wave in feet.											
	300.		350.		400.		450.		500.		600.	
	C.	T.	C.	T.	C.	T.	C.	T.	C.	T.	C.	T.
10	17·7	16·9	—	—	—	—	—	—	—	—	17·9	33·5
50	34·5	8·7	—	—	36·5	11·0	—	—	37·6	13·3	38·3	15·7
100	38·5	7·8	—	—	43·2	9·3	—	—	46·5	10·8	48·8	12·3
1,000	39·1	7·7	42·2	8·3	45·1	8·9	47·9	9·4	50·5	9·9	55·3	10·8
10,000	39·1	7·7	42·2	8·3	45·1	8·9	47·9	9·4	50·5	9·9	55·3	10·8
20,000	39·1	7·7	42·2	8·3	45·1	8·9	47·9	9·4	50·5	9·9	55·3	10·8

Depth of water in feet.	Length of wave in feet.							
	1000.		2000.		5000.		10,000.	
	C.	T.	C.	T.	C.	T.	C.	T.
10	17·9	55·9	—	—	—	—	17·9	557·6
50	39·3	25·4	—	—	—	—	—	—
100	53·2	18·8	—	—	—	—	56·7	176·4
1,000	71·4	14·0	—	—	—	—	168·8	59·2
10,000	71·4	14·0	100·9	19·8	159·6	31·3	225·7	44·3
20,000	71·4	14·0	100·9	19·8	159·6	31·3	225·7	44·3

where γ is the angle between the surface of the water and horizontal plane, T the tangential force, g the gravity, d the depth and ρ the density of the water. COLDING* found that the relation between the wind velocity V and the height to which the water was piled up against the shore by the great storm of November, 1872, in the Baltic could be expressed by the formula

$$V = 14450\sqrt{d \sin \gamma} \qquad (5)$$

in C.G.S. units. Combining the two equations, EKMAN obtained the result

$$T = 0 \cdot 0025 \sigma V^2, \qquad (6)$$

where σ is the density of air. The agreement of this formula with the one obtained by G. I. TAYLOR,† namely,

$$T = 0 \cdot 0023 \sigma V^2, \qquad (7)$$

for the friction of the wind over grass from measurements made on Salisbury Plain is certainly remarkable, and suggests that the above expression does give a fairly accurate value of the frictional force of the wind over a rough surface. It is thus clear from the expression (2) given above for the tangential force required for the maintenance of the sea waves that

$$4\mu\xi pa = 0 \cdot 0025 \sigma V^2. \qquad (8)$$

The theory‡ of maintenance of sea waves and also the recent observations of Dr. VAUGHAN CORNISH§ definitely suggest that the wind velocity exceeds the velocity of the waves by only a small percentage. The calculated values in Table II have been deduced from formulæ (8) and (3), using for the heights of the waves the average observed values and assuming that the velocity of the wind is equal to the velocity of the waves, and, further, that the depth of the ocean is greater than 0·3 km. (1000 feet). The observed values have been taken from KRÜMMEL'S 'Handbuch der Ozeanographie,' vol. 2, p. 73.

We can test the validity of our equations in two ways. First, if wind is capable of exercising a tangential force of the kind we have assumed, the equations should, when the observed values of the amplitude, wave-length and periods are substituted, furnish values for the coefficient of eddy viscosity which should agree with those obtained by

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* 'Danske Vid. Selsk. Skr.,' i, No. 4 (1881).

† 'Roy. Soc. Proc.,' A, vol. 92, p. 196 (1916). Reference is also made to JEFFREYS, 'Phil. Mag.,' vol. 49, p. 801 (1925), where he finds that the coefficient of skin friction for water has the value 0·00256.

‡ JEFFREYS, 'Roy. Soc. Proc.,' A, vol. 107, p. 204 (1925).

§ 'Q.J.R.Met. Soc.,' vol. 52, p. 145 (1926). Dr. VAUGHAN CORNISH'S observations show that for winds concurrent with the direction of waves, the excess of wind velocity, as measured by an anemometer on a ship, over that of the waves is about 10 per cent. Nearer the water surface the wind will be a series of travelling eddies having a smaller velocity, and consequently the excess will be still smaller. Reference is also made to KRÜMMEL'S 'Handbuch der Ozeanographie,' vol. 2, p. 80, where similar results have been given.

altogether different methods. Secondly, when the usually accepted values of the coefficient of eddy viscosity are used in the equations, they should give values for the amplitude of the sea waves agreeing with its average observed values. As regards periods and wave-lengths, we have already noted that equation (3) is in accordance with the observed results. When the first test is applied, the values of the eddy viscosity come out to be those given in the last column of Table II. In order to compare these values with those determined by other methods, we observe that at the discontinuous boundary between air and water, the coefficient of eddy viscosity for air must be the same as that for water, for both must be related in a similar manner to the "skin-friction." The values of eddy viscosity,* so close to the roughened surface of the sea, do not appear to have been determined by any other method, but TAYLOR, from kite observations taken on the *Scotia* during its cruises near Newfoundland, found that the values of μ/σ varied between $0\cdot8 \times 10^3$ and 7×10^3 in C.G.S. units. These show a very good agreement with the figures in the last column of the above table. On the other hand, when μ/σ has these values, the amplitude is found from the equations to have the values given in the last but one column exactly agreeing with those observed. The agreement with the observed values would thus appear to be a sufficient justification for the use of these equations in the present problem.

TABLE II.

Class of sea waves.	Observed values.			Calculated values.				
	Wind velocity.		Height of waves (average) m.	Wave velocity m.p.s.	$\frac{2\pi}{\xi}$ m.	$\frac{2\pi}{p}$ secs.	Height of waves (a) m.	$\frac{\mu}{\sigma}$ C.G.S.
	Beaufort.	m.p.s.						
Very rough .	9	18·0	7·75	18·0	198·2	12·0	7·75	$15\cdot7\times10^3$
Rough . . .	$7\frac{1}{2}$	14·2	5·05	14·2	129·6	9·1	5·05	$7\cdot5\times10^3$
Moderate . .	6	10·7	3·55	10·7	76·3	7·0	3·55	$2\cdot7\times10^3$
Slight . . .	4	6·7	1·60	6·7	30·5	4·4	1·60	$0\cdot6\times10^3$

We therefore obtain the result that wind blowing with a steady velocity V will produce and maintain waves of the type

$$\eta = \frac{0\cdot0025\sigma V^2}{4\mu\xi p} \sin(\xi x - pt), \quad \ldots\ldots\ldots\ldots \quad (9)$$

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* 'Phil. Trans.,' A, vol. 215, p. 21 (1915). Some values for the eddy viscosity of water have been given by JEFFREYS, but these vary within wide limits, and, being based on the assumption that they are independent of depth, are not applicable to the superficial layer, 'Phil. Mag.,' vol. 39, p. 580 (1920). DURST ('Q.J.R. Met. Soc.,' vol. 50, pp. 113–119 (1924)), by a method similar to that of JEFFREYS, obtained the formula $(\mu/\sigma) = 8 \times 10^{-4} V^2$ in C.G.S. units. For a wind of 10 m/s this makes μ/σ equal to $0\cdot8 \times 10^3$ in C.G.S. units. His values, therefore, show a nearer approach to those obtained in Table II.

where p and ξ are connected by the relationship (3) and μ/σ has the appropriate value given in the last column of the above table.

It is not contended that the expressions (9) and (3) represent the actual nature of the surface waves over the Indian Seas during the monsoon months. They are, indeed, very complicated, owing to variations of wind over the different parts, its direction not being concurrent with the direction of waves and other factors. But, certainly, for the purpose of getting numerical estimates of the quantities which we want to verify and for calculating the pressure disturbance at the bottom of the sea, we can assume equations (9) and (3) to be a sufficient approximation to the true condition, particularly during a period when there is a steady wind blowing over the sea. The equations in any case give correctly the observed heights and the periods of the waves for different wind velocities.

7. *Elastic Waves Generated on the Earth's Surface by Sea Waves.*

We can now proceed to obtain mathematical expressions for the microseisms. The gravity waves generated and maintained by the monsoon current on the surface of the sea will produce a disturbance of pressure at its bottom, and this will give rise to forced elastic waves over its bed.

On the assumption that the motion is entirely irrotational, the velocity potential ϕ corresponding to the gravity waves represented by equations (9) and (3) is given by

$$\phi = \frac{0\cdot0025\sigma V^2 g}{4\mu\xi p^2}\cdot\frac{\cosh \xi\,(y+d)}{\cosh \xi d}\cos(\xi x - pt). \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Theory as well as observations suggest that the motion associated with the gravity waves is confined to a superficial layer.* Below this layer the motion depends entirely on the ocean current, which we shall neglect in so far as the present problem is concerned. The equation (10) can therefore at the best be considered to represent the motion very close to the surface for which equation (9) remains approximately true. Over deep water, that is to say, when d is many times larger than the wave-length,

$$\cosh \xi\,(y+d)/\cosh \xi d$$

is approximately equal to unity within the region in which equation (10) may be considered to remain valid, and it therefore reduces to

$$\phi = \frac{0\cdot0025\sigma V^2 g}{4\mu\xi p^2}\cos(\xi x - pt). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

It is important to realise that if equation (10) be considered to represent correctly the

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* JEFFREYS, "On Turbulence in the Ocean," *loc. cit.*

motion right up to the bottom of the sea, we obtain the apparently absurd result that the gravity waves produce almost negligible disturbance of pressure at the bottom of the deep sea. For if equation (10) is true when $y = -d$, the hydrodynamical pressure equation

$$\frac{P}{\rho} = \frac{\partial\phi}{\partial t} - gy - \left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2\right] + \text{const.}$$

gives

$$\frac{P}{\rho} = \frac{g\eta}{\cosh\xi d} + \text{const.}$$

at the bottom of the sea, and the pressure disturbance therefore becomes negligible except when the waves advance over shallow water. This would then suggest that there is no stress disturbance at the bottom of the deep sea, and the microseisms can be developed only when the gravity waves advance over shallow water. But we have already pointed out that microseisms begin to make their appearance long before the arrival of the monsoon current on the Malabar coast with rough seas. Storms also invariably give rise to pronounced microseisms when they are in the mid-Arabian Sea or in the centre of the Bay of Bengal.

When due account is taken of the fact that equation (10) is applicable only to the superficial layer, the pressure disturbance at the bed of the sea can be obtained in two stages. Assume that the motion due to the surface disturbance has become negligible at the depth $y = -h$, where h is only a small fraction of d. Then if P_1 is the pressure disturbance at this depth, equation (11) gives

$$P_1 = g\rho\,(h + \eta).$$

The pressure P_2 due to the column of fluid between this level ($y = -h$) and the bed of the sea ($y = -d$) is simply $g\rho\,(d - h)$. The total pressure at the bed of the sea is therefore $g\rho\,(d + \eta)$. In other words, the stress over each element of area of the bed is equal to the weight of the superincumbent fluid at any instant. That the variation of this total load over each element of area of the bed comes into play in producing elastic waves will be made clear from the subsequent numerical computations, when it will be shown that the amplitudes of the microseisms calculated on this assumption agree with those actually observed. Since the constant part of the stress $g\rho d$ can obviously produce no elastic waves, we shall neglect it and assume that over each unit area of the bed there is at each instant a normal stress of amount $g\rho\eta$.

We assume now that the origin is on the bed of the sea, which for our present problem we shall regard as an infinite plane bounded by $y = 0$, the positive direction of y being now vertically *downwards*. The axis of x is on this plane and is, as before, drawn in the direction of the propagation of waves. Adopting the method given by LAMB* in a paper on the propagation of tremors over the surface of an elastic solid, we notice

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* 'Phil. Trans. Roy. Soc.,' vol. 203, A (1904).

that if u, v are the components of displacements, the elastic equations of motions in two dimensions are satisfied by

$$u = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}, \qquad v = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x}, \quad \ldots \ldots \ldots \quad (12)$$

provided that

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{\lambda + 2\mu}{\delta} \nabla^2 \phi, \qquad \frac{\partial^2 \psi}{\partial t^2} = \frac{\mu}{\delta} \nabla^2 \psi, \quad \ldots \ldots \ldots \quad (13)$$

where (λ, μ) are the usual elastic constants and δ the density of the earth. In the case of simple harmonic motion, the time factor being e^{ipt}, these equations take the forms

$$(\nabla^2 + h^2)\, \phi = 0, \qquad (\nabla^2 + k^2)\, \psi = 0,$$

where

$$h^2 = \frac{p^2 \delta}{\lambda + 2\mu}, \qquad k^2 = \frac{p^2 \delta}{\mu}. \quad \ldots \ldots \ldots \quad (14)$$

A typical solution, applicable to the region $y > 0$, is

$$\phi = A e^{-\alpha y} e^{i\xi x}, \qquad \psi = B e^{-\beta y} e^{i\xi x},$$

where ξ is real, and has the same meaning as before, and α, β are positive real quantities determined by

$$\alpha^2 = \xi^2 - h^2, \qquad \beta^2 = \xi^2 - k^2. \quad \ldots \ldots \ldots \quad (15)$$

These give the following expressions for the displacements (u_0, v_0) and stresses $[X_y]_0$ $[Y_y]_0$ at the plane $y = 0$,

$$u_0 = (i\xi A - \beta B)\, e^{i\xi x}, \qquad v_0 = (-\alpha A - i\xi B)\, e^{i\xi x},$$

and

$$[X_y]_0 = \mu\, \{- 2i\xi\alpha A + (2\xi^2 - k^2)\, B\}\, e^{i\xi x},$$

$$[Y_y]_0 = \mu\, \{(2\xi^2 - k^2)\, A + 2i\xi\beta B\}\, e^{i\xi x}. \quad \ldots \ldots \ldots \quad (16)$$

We see from (9) that, on this plane, the sea waves give rise to the following stresses :—

$$[X_y]_0 = 0, \qquad [Y_y]_0 = g\rho\eta = N e^{i\xi x} \quad \ldots \ldots \ldots \quad (17)$$

where N is a constant,* and is given by

$$N = \frac{0 \cdot 0025 g\rho\sigma V^2}{4\mu\xi p}. \quad \ldots \ldots \ldots \quad (18)$$

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* It is important to note that the quantity μ, which occurs in this constant, is the coefficient of eddy viscosity (see Table II). In all the other expressions in this and the following articles, wherever μ occurs, it denotes rigidity.

The constants A, B are therefore determined by

$$-2i\xi\alpha\mathrm{A}+(2\xi^2-k^2)\,\mathrm{B}=0,$$

$$(2\xi^2-k^2)\,\mathrm{A}+2i\xi\beta\mathrm{B}=\frac{\mathrm{N}}{\mu}.$$

Hence

$$\mathrm{A}=\frac{2\xi^2-k^2}{\mathrm{F}(\xi)}\cdot\frac{\mathrm{N}}{\mu},\qquad \mathrm{B}=\frac{2i\xi\alpha}{\mathrm{F}(\xi)}\cdot\frac{\mathrm{N}}{\mu},\quad \ldots\ldots\ldots\ldots \quad (19)$$

where

$$\mathrm{F}(\xi)=(2\xi^2-k^2)^2-4\xi^2\alpha\beta.\quad \ldots\ldots\ldots\ldots \quad (20)$$

The displacements at any point are given by

$$\left.\begin{aligned} u&=(\mathrm{A}i\xi\, e^{-\alpha y}-\mathrm{B}\beta\, e^{-\beta y})\, e^{i\xi x}\\ v&=(-\alpha\mathrm{A}\, e^{-\alpha y}-i\xi\mathrm{B}\, e^{-\beta y})\, e^{i\xi x}\end{aligned}\right\},\quad \ldots\ldots\ldots\ldots \quad (21)$$

and taking only the real part, by

$$\left.\begin{aligned} u&=-\frac{\mathrm{N}}{\mu}\left\{\frac{2\xi^2-k^2}{\mathrm{F}(\xi)}\,\xi\, e^{-\alpha y}-\frac{2\xi\alpha}{\mathrm{F}(\xi)}\,\beta\, e^{-\beta y}\right\}\sin\xi x\cos pt\\ v&=\frac{\mathrm{N}}{\mu}\left\{-\frac{2\xi^2-k^2}{\mathrm{F}(\xi)}\,\alpha\, e^{-\alpha y}+\frac{2\xi\alpha}{\mathrm{F}(\xi)}\,\xi\, e^{-\beta y}\right\}\cos\xi x\cos pt\end{aligned}\right\},\quad \ldots \quad (22)$$

which represent a set of standing vibrations on the bed of the sea combining to form progressive waves. Equation (17) shows that the periods and wave-lengths of these forced waves are exactly the same as those of the sea waves which generate them. They are therefore given by

$$p^2=g\xi\tanh\xi d.\quad \ldots\ldots\ldots\ldots \quad (23)$$

There will be resonance and the solution will become infinite, if $\mathrm{F}(\xi)=0$, that is to say, if $(2\xi^2-k^2)^2-4\xi^2\alpha\beta=0$, which, on eliminating α, β by (15), reduces to

$$1-8\frac{\xi^2}{k^2}+\left(24-16\frac{h^2}{k^2}\right)\frac{\xi^4}{k^4}-16\left(1-\frac{h^2}{k^2}\right)\frac{\xi^6}{k^6}=0.\quad \ldots\ldots \quad (24)$$

This is the cubic equation which occurs in the investigation of RAYLEIGH waves. We shall show that these forced waves can never satisfy the above equation for free vibrations and therefore there can be no resonance.

When POISSON's hypothesis as to the relation between the elastic constants is satisfied, namely, when $\lambda=\mu$, we get $h^2=\frac{1}{3}k^2$. The three roots of the above equation are then

$$\frac{\xi^2}{k^2}=\tfrac{1}{4},\qquad \tfrac{1}{4}(3-\sqrt{3}),\qquad \tfrac{1}{4}(3+\sqrt{3}).$$

As we are now investigating waves which do not penetrate far into the interior of the earth, α, β must be positive real quantities. Only the last root, namely,

$$\xi^2/k^2 = \tfrac{1}{4}(3 + \sqrt{3}),$$

will make them so. We thus get, if c is the velocity of propagation of Rayleigh waves,

$$c = \frac{\text{wave-length}}{\text{period}} = p/\xi.$$

But

$$k^2 = \frac{p^2\delta}{\mu} = \frac{c^2\xi^2\delta}{\mu}.$$

Therefore

$$\left.\begin{array}{c} c = \dfrac{k}{\xi}\sqrt{\dfrac{\mu}{\delta}} = 0\cdot9194\sqrt{\dfrac{\mu}{\delta}} = 3\cdot0 \text{ km. per second} \\ \text{(for the upper layer of the earth's crust)} \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \qquad (25)$$

Assuming that the depth of the ocean exceeds 1000 feet, or 0·3 km., it will be seen from Table I that for the range of periods, namely, 4 to 10 seconds, usually met with in the monsoon microseisms, the wave-length $2\pi/\xi$ of the waves produced on the sea will be less than 600 feet, or 0·2 km. For these periods the wave-lengths of the Rayleigh waves will be greater than 12 km. The equation $F(\xi) = 0$ cannot, under the circumstances, be satisfied by wave-lengths of the order 0·2 km., and the solution (22) given above cannot therefore be infinite.

It is obvious that on the solution (22) we can superpose a system of free surface waves, namely, the Rayleigh waves, having the wave-length proper to the imposed period $2\pi/p$, without in any way vitiating the surface conditions.

We can compare the behaviour of these forced waves with that of Rayleigh waves in relation to the constitution of the earth's crust, which according to seismological evidence is supposed to consist of two or three distinct layers. Since the periods and wave-lengths of the forced waves are given by $p^2 = g\xi \tanh \xi d$, and $\tanh \xi d$ is very nearly equal to unity for deep water, we see from equations (14) and (15) that

$$\alpha^2 = \xi^2 - \frac{g^2\delta}{\lambda + 2\mu}\xi^2, \qquad \beta^2 = \xi^2 - \frac{g^2\delta}{\mu}\xi^2.$$

Now the average values of $\sqrt{\dfrac{\lambda + 2\mu}{\delta}}$ and $\sqrt{\dfrac{\mu}{\delta}}$ for the earth's crust are nearly 6·2 km. per second and 3·7 km. per second respectively, and therefore $g^2\delta/(\lambda + 2\mu)$ and $g^2\delta/\mu$ are each of the order 10^{-5}. We can therefore take α and β to be each equal to ξ. These values of α and β may be compared with those of the corresponding quantities

(r, s) which occur in the following expressions for RAYLEIGH waves, when POISSON'S condition is satisfied.

$$u = \mathrm{A}\,(e^{-ry} - 0\cdot5773\,e^{-sy}) \sin\kappa\,(x - ct),$$
$$v = \mathrm{A}\,(0\cdot8475\,e^{-ry} - 1\cdot4679\,e^{-sy}) \cos\kappa\,(x - ct),$$
$$r = 0\cdot8475\kappa, \qquad s = 0\cdot3933\kappa,$$

where $2\pi/\kappa$ is the wave-length of RAYLEIGH waves. It would thus appear that these forced waves, like RAYLEIGH waves, will penetrate a depth which is proportional to the wave-length. But whereas RAYLEIGH waves generated by earthquake shocks have generally periods of the order of 20 seconds and wave-lengths of 60 km., the forced waves, which will be produced by a monsoon current having a velocity of 25 miles per hour, will have periods of 7 seconds and wave-lengths, as given by equation (23), of 76 metres only. While, therefore, the RAYLEIGH waves, particularly those of long wave-lengths, will penetrate deep into the lower layer of the earth's crust, these forced waves will be confined mainly on the surface of the upper layer.

The solution (22) is, of course, valid for the region over which the normal stress $\mathrm{N}e^{i\xi x}$ is acting, that is to say, over the bottom of the sea wherever the monsoon is active. When these forced vibrations arrive over the land surface or over a region where the normal stress is zero, they will give, as a consequence of this surface condition, rise to such free vibrations of RAYLEIGH type as are caused by an initial periodic disturbance, and these will travel with the velocity of propagation of RAYLEIGH waves. As the initial condition will impose definite periods, the RAYLEIGH waves so generated will maintain these periods but will increase in wave-length. If the waves on the surface of the sea are maintained by a steady wind, the tremors at its bottom will also be maintained. Within the region of disturbance there will be no diminution of amplitude with distance, but when these tremors travel as RAYLEIGH waves over a tract of country, with no stress on its surface, the amplitude will undergo diminution with distance owing to dissipation, according to the usual law of diminution of RAYLEIGH waves. The observed law of diminution of amplitude of microseisms with distance has already been shown in figs. 5 and 6, and this suggests a decrement according to the usual exponential law as in the case of other dissipative forces. In the case of a storm raging over the sea the disturbance will be confined within an area of 100 to 200 square miles, and an observing station will record microseisms travelling from this disturbed region with the velocity of propagation of RAYLEIGH waves. This being of the order of 3 km. per second for the superficial layer, it should be possible for an observing station situated within a few hundred miles from a storm to record its existence as soon as it begins to disturb the sea.

8. *The Calculated and the Observed Periods of Microseisms.*

The variations of periods and wave velocities for different depths of the sea and different wave-lengths, which are governed by equation (23), have already been given in Table I. A reference to fig. 7 will lead to a clear understanding of the laws of variations of the periods. It has already been remarked that both theory and observations show that for maintained waves the velocity of wind will exceed the velocity of the waves by a small percentage, and for our present discussion we can assume them to be equal. The depth of the Arabian Sea and the Bay of Bengal exceeds 1000 feet, except for a few miles near the coast, and we thus see from the curve that a wind velocity of 20 miles per hour will produce periods of about 6 seconds, a wind velocity of 40 miles per hour will produce periods of about 11 seconds, and a wind velocity of 60 miles per hour will produce periods of about 17 seconds.

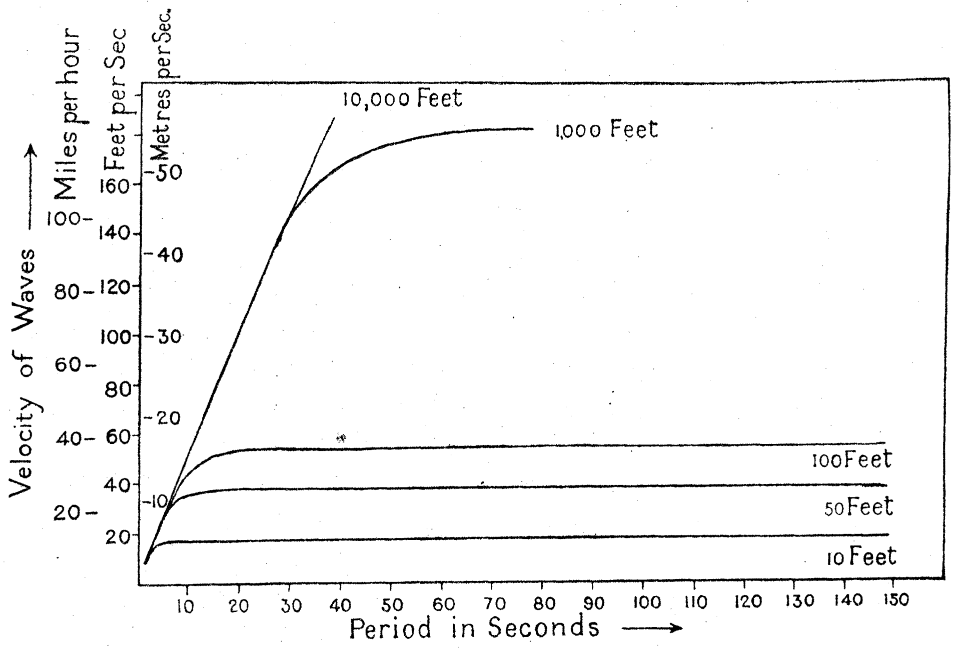

FIG. 7.

From fig. 4 it will be seen that the average (monthly) periods of microseisms during the monsoon months vary between 6 to 7 seconds, and these will be produced by wind velocity of 20 to 25 miles per hour. A reference to DALLAS's 'Meteorological Atlas of the Indian Seas' will show that the average wind force over the Indian Seas during these months is 5, that is to say, about 19 to 24 miles per hour. The average periods of microseisms therefore agree with those which will be produced by the average velocity of the monsoon wind over the sea areas.

For lack of observations, the determination of the value of the mean wind velocity over the sea areas on individual days and the comparison of the observed periods of microseisms with the theoretical values is a matter of some difficulty. The results of pilot balloon observations suggest that over ordinary plain grounds the gradient velocity is attained at a height of 0·3 km., and over rugged grounds at a height of 0·5 km. or more. Over sea the gradient velocity is attained at a lower height* between

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* 'Phil. Trans.,' A, vol. 215, p. 21 (1915); also 'Manual of Meteorology,' Part IV, pp. 20, 29.

0·1 km. to 0·3 km. It is also known that over the sea the surface wind velocity is approximately 2/3 of the gradient velocity, at a coastal observatory it is $\frac{1}{2}$, and at an inland station it is 1/3. I have checked the pilot balloon observations at Bombay and have found that they satisfy the approximate relation stated above. The ratio of 2/3 for the sea is confirmed by the theoretical curve given by G. I. TAYLOR on p. 19 of 'Phil. Trans. Roy. Soc.,' A, vol. 215 (1915).

By counting the number of isobars over the sea areas on individual days and assuming a uniform gradient, which is roughly true during a steady monsoon, the gradient velocity* is easily calculated, and from these the surface wind velocity can be deduced by the above law. A calculation of the gradient velocity from the distribution of normal isobars over the Indian seas during the months, June, July, August and September, as given in DALLAS's 'Meteorological Atlas of the Indian Seas,' gives the following results :—

	Normal gradient wind velocity (in miles per hour).	Normal surface wind velocity (in miles per hour).
June	41·7	19
July	48·6	23
August	39·0	21
September	26·9	13

From this it will be seen that in these months the ratio of the surface velocity to gradient velocity is 1 : 2 and not 2 : 3. This may be partly due to the fact that the average direction of normal monsoon isobars has been taken to be along great circles, which is only approximately true, and partly to the fact that, in these months, the sea becomes rough and consequently the frictional force is greater than that of a surface over which the ratio 2 : 3 is applicable. For individual days also, then, we shall assume the ratio to be 1 : 2 and not 2 : 3. In the following table (No. III) the calculated and the observed periods of microseisms have been given for some dates selected at random. The observed surface wind velocity has been obtained from the gradient velocity on the different dates in the manner explained above from the 8 hours' weather charts.

The observed periods no doubt show a general increase with increase in wind velocity, but they are smaller than the calculated periods, particularly for high winds. We have, however, to note that an oscillatory system like the sea, which has established itself to one particular period, cannot change its period suddenly. The essential condition under which the periods of sea waves can change to a higher value when there is an increase in wind velocity is that this increased velocity should be maintained for a fairly

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* See 'M.O. Computer's Handbook,' Section II, art. 4, pp. 71–74. The figures for a mean latitude of 12° were obtained by fresh calculation and not by extrapolation. The average direction of the normal monsoon isobars over the Arabian Sea is along great circles, and so the cyclostrophic component is negligible. Over the Bay of Bengal the isobars curve round small circles under the influence of the Himalayan Range and Burma Hills.

TABLE III.

Date.	Gradient wind velocity from the distribution of isobars over sea areas (in miles per hour).	Surface wind velocity calculated from gradient velocity (in miles per hour).	Observed periods of microseisms (in seconds).	Calculated periods of microseisms (in seconds).
July 4, 1924	49·3	25	6·0	7·7
June 5, 1927	52·1	26	6·0	
June 2, 1925	53·5	27	5·5	
July 18, 1925	53·5	27	6·0	
May 26, 1925	53·5	27	5·6	
July 25, 1926	54·7	27	6·0	
July 16, 1925	54·7	27	6·0	
July 7, 1926	56·0	28	6·0	
August 12, 1927	56·9	28	6·7	
June 17, 1924	62·5	31	7·1	9
July 10, 1927	67·4	34	7·1	
June 30, 1925	67·4	34	7·6	
June 25, 1925	79·7	40	7·5	11
June 28, 1925	93·2	47	7·5	

long time so that a new steady condition is established. This can seldom happen, for the configuration of the isobars, although maintaining the usual monsoon characteristics, shows fluctuations from day to day. The tendency of the sea waves will therefore be to maintain an average period corresponding to the average wind velocity. They will show a slight increase in period when there is a temporary increase in the wind velocity, but will never attain the full period corresponding to the particular velocity unless it is maintained for a long time so as to establish waves of its own period.

If the velocities of the wind over two different regions of the sea surface show a marked difference, they will be effective in producing waves of two distinct periods, which will eventually superpose. Let us denote the two regions by the suffixes 1 and 2, and assume that the velocity of the wind over the first region is greater than that over the second. If

$$\eta_1 = A_1 \cos(\sigma_1 t - \varepsilon_1), \qquad \eta_2 = A_2 \cos(\sigma_2 t - \varepsilon_2)$$

represent the wave systems generated over the two regions, then since the waves over the first region will have longer periods than those in the second we must have $\sigma_1 < \sigma_2$. On the other hand, equation (9) shows that A_1 will be greater than A_2.

The superposed oscillations will therefore be of the well-known type,

$$\eta = A_1 \cos(\sigma_1 t - \varepsilon_1) + A_2 \cos(\sigma_2 t - \varepsilon_2) = C \cos(\sigma_1 t - \varepsilon_1 - \alpha)$$

where

$$C = (A_1^2 + 2A_1A_2 \cos\phi + A_2^2)^{\frac{1}{2}}, \quad \alpha = \tan^{-1} A_2 \sin\phi/(A_1 + A_2\cos\phi),$$

$$\phi = (\sigma_1 - \sigma_2)\,t - \varepsilon_1 + \varepsilon_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

It is clear that if the wind velocities over the two regions differ only by a small quantity, then the oscillations will have periods corresponding to the component waves of larger periods, but both the amplitude and the phase will undergo slow variation. The variations in the amplitude and phase of the observed monsoon microseisms during a period of active monsoon are so small that either the differences in the wind velocities over different parts of the sea are inappreciable, and even if appreciable they last for such a short time that they are not effective in producing waves differing markedly in periods, or else the minor complications introduced in the superposed waves are wiped out, during their travel to the observing station, by dissipation. When the monsoon advances with a depression in front there is a marked difference in the wind velocities in the region of depression and the monsoon area to its south. In such cases the microseisms invariably show well-pronounced variations, both in amplitude and phase. The dissipative forces are therefore not powerful enough in smoothing out the microseisms when there is a marked difference in the wind velocity lasting for 1 or 2 days or more over the different parts of the sea, and the conclusion is, therefore, irresistible from the observed microseisms that unless a depression or a storm has formed there is not much difference in the mean wind velocity over the main parts of the sea, and even if there is any it does not last long to be effective in producing waves of different periods, a conclusion which is also supported by the uniform configuration of the normal isobars* during the monsoon months.

A variation in the amplitude of the microseisms will, however, be caused by a slow variation in the velocity of the wind prevailing over the sea areas. In fact, it will be shown later in this paper that the amplitude of the long-period microseisms, which are caused by waves over the shallow sea near the coast during pronounced land and sea breezes, undergoes a diurnal variation analogous to that of the wind.

A case involving superposition of waves of different periods which calls for special treatment is that of microseisms associated with storms in the Bay of Bengal and the Arabian Sea. In these storms, the wind velocity in the storm area will differ widely from that in the surrounding regions. A severe storm of this kind will generally have a ring of hurricane winds in which the wind velocity will be 75 miles per hour or more (110 feet per second or more), included roughly between two circles of 20 and 75 miles radii. The storm may be considered to be embedded in a flowing current covering a wide area in which the velocity will be about 20 miles per hour. The hurricane winds in the storm area will produce waves having periods of about 22 seconds or more, whereas the outlying winds will produce waves having periods of 6 seconds. It is clear from equation (26) that when the periods of the component waves differ so widely from each other, the resultant movements will not have the characteristics of either of the component waves. The energy communicated to the surface of the earth for the production of the tremors not only depends on the heights of the waves but also on the area of the disturbed regions. The disturbance due to the storm being confined to a

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* W. L. Dallas, ‘Meteorological Atlas of the India Seas.’

small area, the energy contributed by it will be comparable with that contributed by winds over the surrounding regions. The two terms

$$A_1 \cos(\sigma_1 t - \varepsilon_1) \quad \text{and} \quad A_2 \cos(\sigma_2 t - \varepsilon_2)$$

will therefore be of equal importance, and consequently the resultant movements will show small period waves superposed on those of the large periods with considerable prominence. In fig. 8 has been shown, on an open scale, the composition curve of two simple harmonic waves in the same phase and having equal amplitudes, but periods of 20 seconds and 5 seconds respectively. The curve vividly presents to the eye how the resultant movements due to two such systems of waves have periods corresponding to that of the small period waves. It is important to bear this familiar principle in mind because the periods of the storm microseisms are pre-eminently those of the small period waves produced by the outlying winds. On the other hand, a cyclonic circulation will produce waves directed towards all the points of the compass, which must assume very complicated forms as the result of interference between themselves, and with the small period waves produced by the winds in which it is embedded. The effect of a random distribution of such waves of two distinct periods, moving in all possible directions and interfering with each other, will be to produce irregular variations in amplitude, and these variations will on the average have a period corresponding to that of the large period waves. It is worth while noticing that the average period of the variations of the amplitudes of the microseisms reproduced in Plate 1 is about 20 seconds, and that this will agree roughly with the periods which will be produced by winds of storm force.

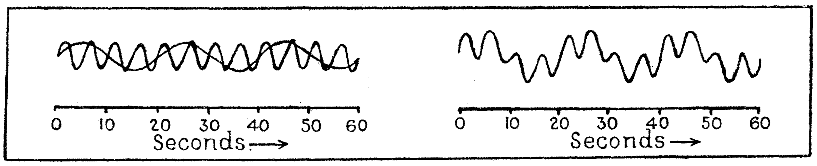

FIG. 8.

It is well known that the winds in tropical storms are squally in character, coming in puffs at intervals of a minute or two and attaining high velocities. It is difficult to estimate whether this factor plays any part in producing variations in amplitude. We have, however, seen that water waves have a tendency to settle down to a period corresponding to that of the mean wind velocity, and do not respond to quick fluctuations in the velocity.

9. *The Calculated and the Observed Amplitudes of Microseisms.*

Equation (22) gives the forced elastic waves produced at the bottom of the sea by the water waves. Now since the periods and wave-lengths of these waves are connected by the relationship $p^2 = g\xi \tanh \xi d$, the velocity of propagation of these forced waves within the area in which they are confined must be the same as that of the water waves.

Now if V is the velocity of propagation of these forced waves, then $V = p/\xi$, but $k^2 = p^2\delta/\mu$, and therefore

$$V = \frac{k}{\xi}\sqrt{\frac{\mu}{\delta}}.$$

Let us assume V, which is equal to or slightly less than the velocity of wind, to be equal to 10 metres per second, for the purpose of getting numerical estimates of the amplitudes of the earth movements. Now $(\mu/\delta)^{\frac{1}{2}}$ is the velocity of propagation of transverse elastic waves in the upper layer of the earth's crust, and this may be taken, as given by the observations of earthquakes, to be equal to 3·3 km. per second.

We thus get

$$\frac{k}{\xi} = \frac{10}{3\cdot 3 \times 10^3} = \tfrac{1}{330},$$

and this is a very small fraction. Similarly, h/ξ is found to be $= \frac{1}{550}$ in the upper layer.

We have therefore

$$\begin{aligned} F(\xi) &= (2\xi^2 - k^2) - 4\xi^2\alpha\beta, \\ &= (2\xi^2 - k^2)^2 - 4\xi^2(\xi^2 - k^2)^{\frac{1}{2}}(\xi^2 - h^2)^{\frac{1}{2}}, \\ &= 4\xi^4\left(1 - \frac{k^2}{\xi^2}\right) - 4\xi^4\left(1 - \frac{1}{2}\frac{k^2}{\xi^2}\right)\left(1 - \frac{1}{2}\frac{h^2}{\xi^2}\right), \\ &= 2\xi^2(h^2 - k^2), \text{ neglecting small quantities} \quad \ldots\ldots \quad (27) \end{aligned}$$

On Poisson's hypothesis $\lambda = \mu$, and therefore $h^2 = \frac{1}{3}k^2$. Hence we get

$$\left.\begin{aligned} \frac{2\xi^2 - k^2}{F(\xi)} &= \frac{2\xi^2\left(1 - \dfrac{k^2}{2\xi^2}\right)}{2\xi^2(h^2 - k^2)} = \frac{1}{h^2 - k^2} = -\frac{3}{2k^2} \\ \frac{2\xi\alpha}{F(\xi)} &= \frac{2\xi(\xi^2 - h^2)^{\frac{1}{2}}}{2\xi^2(h^2 - k^2)} = \frac{1}{h^2 - k^2} = -\frac{3}{2k^2} \end{aligned}\right\} \quad \ldots\ldots \quad (28)$$

If u_0, v_0 denote the displacements due to the forced waves on the bed of the sea, $y = 0$, then from (22) and (28) we get

$$\left.\begin{aligned} u_0 &= \frac{N}{\mu}\cdot\frac{3}{2k^2}(\xi - \beta)\sin\xi x\cos pt \\ v_0 &= -\frac{N}{\mu}\cdot\frac{3}{2k^2}(\xi - \alpha)\cos\xi x\cos pt \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (29)$$

Now

$$\left.\begin{aligned} \xi - \beta &= \xi - (\xi^2 - k^2)^{\frac{1}{2}} = \xi - \xi\left(1 - \tfrac{1}{2}\frac{k^2}{\xi^2}\right) = \tfrac{1}{2}\frac{k^2}{\xi} \\ \xi - \alpha &= \xi - (\xi^2 - h^2)^{\frac{1}{2}} = \xi - \xi\left(1 - \tfrac{1}{2}\frac{h^2}{\xi^2}\right) = \tfrac{1}{2}\frac{h^2}{\xi} = \tfrac{1}{6}\frac{k^2}{\xi} \end{aligned}\right\} \quad \ldots\ldots \quad (30)$$

Therefore

$$\left.\begin{aligned} u_0 &= \frac{\mathrm{N}}{\mu}\cdot\frac{3}{4\xi}\sin\xi x\cos pt \\ v_0 &= -\frac{\mathrm{N}}{\mu}\cdot\frac{1}{4\xi}\cos\xi x\cos pt \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (31)$$

We thus get the interesting result that *the amplitude of the horizontal displacement is three times the amplitude of the vertical displacement, and that the phases of the two movements are at right angles to each other, the motion of each particle being elliptic.* This relationship is true at the bottom of the sea. Over a region where the normal stress is zero, these waves, as already explained, will travel as free surface waves and the amplitudes both of the horizontal and vertical components will undergo diminution with distance according to an exponential law. At a coast station, very near the disturbed sea, the ratio of the amplitude of the horizontal to that of the vertical component will be nearly 3 : 1, while at an inland station, far away from the coast, where the waves have assumed the full characteristics of RAYLEIGH waves, the ratio will be the usual one for RAYLEIGH waves, namely, 0·7 : 1. We can thus conclude, on theoretical grounds, that the maximum value of this ratio is 3 : 1, and the minimum value 0·7 : 1, the exact value at a station depending on its distance from the disturbed region.

If the depth of the sea exceeds 1000 feet, then we see from Table I that the period of the waves, which travel with a velocity of 10·7 metres per second, is about 7 seconds, and $2\pi/\xi$ = wave-length = 7630 cms. Now the rigidities of many kinds of granite and marble were found by ADAMS and COKER to be roughly about $2\cdot5 \times 10^{11}$ dynes per square centimetre. For numerical computation, let us assume that the rigidity* of the earth's crust is given by $\mu = 3 \times 10^{11}$ dynes per square centimetre, and therefore $(\mu/g) = 3 \times 10^8$. From Table II it is seen that the average elevation of sea waves produced by wind blowing with a velocity of 10·7 metres per second is 3·55 metres.

Therefore

$$\mathrm{N}/\mu = g\rho\eta/\mu = g\eta/\mu = 355/3 \times 10^8 = 118 \times 10^{-8},$$

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* DARWIN has shown that the effective rigidity of the earth is at least as great as that of steel, 'Scientific Papers,' vol. 1, p. 346 ; also p. 448. Taking 2·7 for the density and 3·3 km. per second for the S wave velocity in the upper layer of the earth's crust, μ comes out to be 3×10^{11} dynes per square centimetre.

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taking ρ, the density of water, to be equal to unity. We thus get

$$\left.\begin{aligned} u_0 &= (12 \times 10^{-3}\text{ mm.}) \sin \xi x \cos pt, \\ v_0 &= -(4 \times 10^{-3}\text{ mm.}) \cos \xi x \cos pt \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (32)$$

It will thus be seen that a wind velocity of 10·7 metres per second will produce microseisms, in which the average amplitude of the horizontal displacement will be of the order of 12 microns, and that of the vertical displacement of the order of 4 microns. The amplitudes of microseisms calculated in the manner explained above, for various classes of sea waves, are given in the following table :—

Table IV.

Class of sea waves.	Wind velocity.		Wave-length.	Height of sea waves.			Calculated amplitudes of microseisms (in microns).					
	Beaufort.	m.p.s.		Average.	Maximum.	Minimum.	Average.		Maximum.		Minimum.	
							u_0.	v_0.	u_0.	v_0.	u_0.	v_0.
			m.	m.	m.	m.						
Very rough .	9	18·0	198·2	7·75	11·5	6·5	63	21	93	31	51	17
Rough . . .	7½	14·2	129·6	5·05	7·5	3·5	27	9	39	13	18	6
Moderate . .	6	10·7	76·3	3·55	6·5	2·3	12	4	21	7	6	2
Slight . . .	4	6·7	30·5	1·60	4·0	0·8	2	0·7	6	2	0·9	0·3

The heights of the waves given in the above table have been taken from Krümmel's Handbuch der Ozeanographie,' vol. 2, p. 73, and they agree with those given by equation (9).

The calculated values given in the above table give the amplitudes of the microseisms produced at the bottom of the sea by the water waves. When travelling over an undisturbed region the amplitudes will undergo diminution with distance, and consequently the amplitudes recorded at an observing station will be of much smaller order than those given in the above table, according to its distance from the disturbed region. The fact that the amplitude of the microseisms recorded in the Colaba Observatory on November 12, 1927, became 8 microns, when a storm was crossing coast between Bombay and Ratnagiri some 30 miles south of Bombay, certainly testifies to the correctness of the figures given in the above table for the disturbed region. The law of diminution of amplitude during the travel of these microseisms over an otherwise undisturbed region can be taken, for all practical purposes, to be as given in fig. 5.

The observed amplitudes of microseisms at Bombay in the different months of the year are given in the following table :—

TABLE V.—Amplitude of Microseisms (N—S) in Microns.

Month.	1924.		1925.		Month.	1924.		1925.	
	Average.	Maximum.	Average.	Maximum.		Average.	Maximum.	Average.	Maximum.
January .	1·0	2·3	1·4	3·4	July . .	0·9	1·7	0·8	1·3
February	0·9	2·3	1·6	4·1	August .	0·7	1·7	0·6	1·0
March . .	0·8	3·4	1·7	4·3	September	0·5	1·0	0·7	1·3
April . .	0·7	2·9	0·9	2·8	October .	0·8	1·7	0·8	1·7
May . .	0·6	1·0	1·0	2·1	November	0·7	1·5	0·7	1·4
June . .	0·9	1·4	0·8	1·5	December	0·9	2·9	0·9	4·0

The quantities in Table V are such as can be easily explained from the calculated values, given in the previous table, reduced on account of the distance of the Observatory from the disturbed region in the sea.

The Colaba Observatory has no vertical seismograph, and I have therefore not been able to make a direct verification of the theoretical result that the ratio of the amplitude of the horizontal movements to that of the vertical movements varies from 3 to 0·7 according to the distance of the disturbed region. I have, however, examined some of the published Galitzian seismograms of other Observatories which were available to me, and I find that the vertical component is much quieter than the horizontal component, and invariably shows microseisms of smaller amplitude. As regards published observations,* ZÖPPRITZ finds that at Göttingen, the amplitude of the horizontal component to that of the vertical component has a maximum value of 3 and a minimum value of 0·5. Out of 436 measurements of the records obtained at Pulkovo, MAINKA found that in 66 the amplitude of the horizontal component was less than that of the vertical component, in 316 it was greater, in 54 the amplitudes were equal, and in 23 the ratio of the amplitude was greater than 2. GUTENBERG finds from the observations taken during the years 1906–10 at this station that the ratio is on the average 1·7. He has also given, as the result of tabulation for 2 hours on every day of records obtained for 5 days at the undermentioned stations, the average ratio mentioned against their names :—

Göttingen . .	1·6	Pulkovo . . .	1·1	Strasbourg .	2·8
Baku	2·3	Bochum . . .	2·0	Ekaterinenburg	1·1
Tiflis	2				

The records of the Galitzin seismographs (horizontal and vertical components) at the Eskdalemuir Observatory for September 13, 1912, on which date the Dardanelles earthquake occurred, and also for the 14th, have been reproduced (unreduced in scale) in WALKER'S 'Modern Seismology.' After the end of the disturbance due to the

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* GUTENBERG, "Die Seismische Bodenunruhe," 'Sammlung geophysikalischer Schriften,' p. 33 (1924).

shock, the records show fine microseisms for several hours and these have been tabulated in Table VI. In deducing the values in microns, the magnification of the instruments was calculated from their published constants, by the method explained on pp. 332–334 of the 'Observatories' Year Book,' 1926 (London Meteorological Office). The magnification of the horizontal pendulums for both N-S and E-W components was found to be 345 times and of the vertical component 373 times. In the following table the horizontal component of the microseisms in the N-S direction is denoted by A_N, in the E-W direction by A_E, and in the vertical direction by A_Z.

TABLE VI.

1912 (hours).	A_N.	A_E.	$(A_N^2+A_E^2)^{\frac{1}{2}}$.	A_Z.	$(A_N^2+A_E^2)^{\frac{1}{2}}/A_Z$.
h. m.	μ	μ	μ	μ	
September 13, 23 0	4·1	2·9	5·0	1·6	3·1
23 30	2·6	2·3	3·5	1·2	2·9
September 14, 1 30	3·5	2·9	4·6	1·6	2·9
2 0	2·9	2·9	4·1	1·5	2·7
2 30	3·2	2·9	4·4	1·5	2·9
3 0	3·5	2·9	4·6	1·2	3·8
3 30	2·9	2·6	3·9	1·3	3·0
4 0	3·5	2·9	4·6	1·3	3·5
				Mean . .	3·1

The published observations, as well as the records available to me, appear therefore to support strongly the law deduced from a purely theoretical consideration.

We can thus conclude that the theory of microseisms as explained in this paper is satisfactorily confirmed by the observed results.

10. *Long Period Microseisms.*

There is another type of microseisms which make their appearance in Colaba records on days of pronounced land and sea breezes during the dry months, November to May. The periods of these movements range from 10 seconds to 30 seconds. It has been my practice during the last few years to examine the records day after day for microseisms in relation to weather conditions. Under the adjustment adopted for the MILNE-SHAW seismograph, these microseisms were very feebly marked in the records when the wind velocity as recorded in the Observatory was 15 to 20 miles per hour, and for wind velocity of less than 15 miles they were almost invisible. On the other hand, when the winds at Bombay exceeded this value, they were invariably present and quite pronounced. The duration of these long-period microseisms, their average periods, amplitudes, as well as the duration of the average velocity of wind above 20 miles per hour, during the years 1924–28, were arranged in the form of a table, as shown in Appendix B for one of the years, and this made the mutual relationship* between the local winds and these large period microseisms obvious. The microseisms were found to start with the wind, increase in intensity as the wind velocity increased and disappear as the wind subsided. In every case the amplitudes of the microseisms were found to follow the diurnal variation of the wind. A few typical cases during the current year have been plotted in fig. 9. Appendix B shows that most of the long-period microseisms are associated with the sea breeze, and this is due to the fact that at Bombay, as in many other coast stations, the velocity and the regularity of the sea breeze are much more marked than is the case with the land breeze. The latter seldom reaches a velocity of 20 miles per hour.

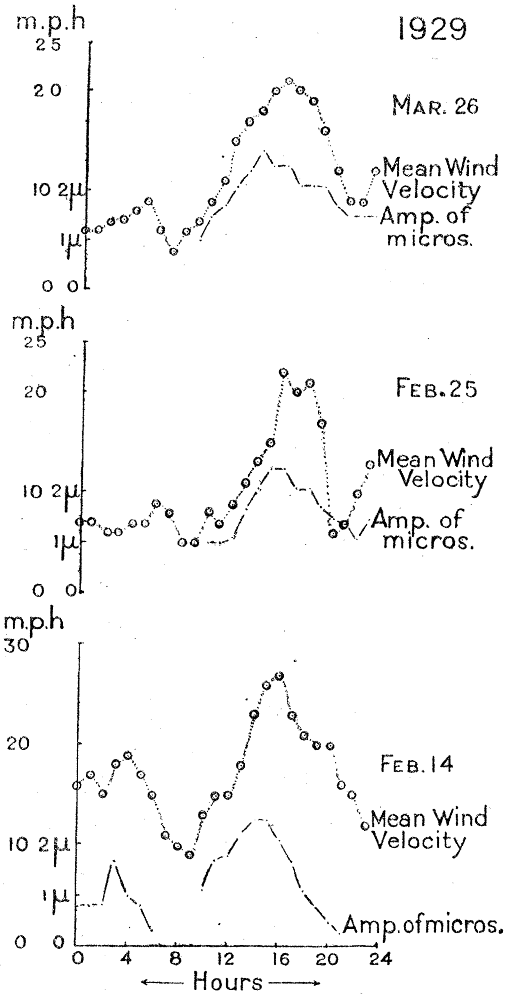

FIG. 9.

It has been supposed by some seismologists† that these large period movements are due to the gusts of the local wind setting the trees and buildings into movement. However plausible such an argument might seem to be superficially, it would seem to be untenable when the dynamical consequences of the movements of trees and buildings are compared with the observed movements. The theory of lateral vibrations of bars is not strictly applicable to an ordinary building, which, for the sake of simplicity, we shall assume to be a solid masonry wall, cubical in shape, but can be taken to be true as a first approximation when the terms depending on the rotatory inertia are taken into account. If the wall is assumed to be of height l, length a, and breadth b, and, if further, the wind blows perpendicularly to its face and one principal axis lies in the plane of vibration, then the frequency p of its gravest mode of vibrations‡ is given by

$$p = \left(1 - 2\cdot3241\,\frac{K^2}{l^2}\right)\frac{KV}{2\pi l^2}m^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

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* The duration of microseisms is in most cases longer than the duration of the average velocity of wind above 20 miles per hour, and this is due to the fact that feeble microseisms appear even when the velocity is lower than 20 miles per hour, and the tabulation has been made from the point where they became just visible up to the point where they disappeared.

† WALKER, 'Modern Seismology,' p. 73.

‡ RAYLEIGH, 'Theory of Sound,' vol. 1, arts. 171, 186.

in which V is the velocity of progagation of longitudinal waves, depending only on the material of which the building is made, K the radius of gyration of any transverse section about the axis through its centre of inertia perpendicular to the plane of bending, and m is a root of the equation $\cos m \cosh m + 1 = 0$ and has the value $1 \cdot 875$ for the gravest mode. We have $K^2 = \frac{1}{12} b^2$, and taking V = velocity of propagation of longitudinal waves = 5·5 km. per second and $l = 4b = 20$ metres, and substituting in equation (33), we find that the wall will execute 22 vibrations per second in the gravest mode.

A seismograph of the Omori type was suspended from one of the walls of the Tower in the Colaba Observatory, which is a stone structure of cylindrical shape, rising to a height of about 40 feet above the ground. The instrument was allowed to work for about 2 years, during which the records were almost always disturbed by small vibrations, having periods practically equal to the free periods of the pendulum, which varied, from time to time, between 20 to 30 seconds. This indicates that the quakes in the walls, caused by the gusts of wind or other causes, were continually subjecting the pendulum to execute small free oscillations of its own. It never recorded the free oscillations of the Tower; in fact, they were so quick that it could not. Theoretically, the free periods* of vibrations of a cylinder of radius a, length $2l$ and thickness $2h$, which is vibrating by pure bending, are given by

$$p_s^2 = \tfrac{4}{3}\,\frac{mn}{m+n}\,\frac{h^2}{\delta a^4}\,\frac{(s^3-s)^2}{s^2+1}\cdot\left\{1+\frac{3a^2}{s^2l^2}\cdot\frac{m+n}{m}\right\}\Big/\left\{1+\frac{3a^2}{(s^4+s^2)\,l^2}\right\}, \quad . \; . \; . \quad (34)$$

where $2\pi/p_s$ denotes the period, the different modes of vibrations being given by $s = 2, 3, 4$, etc. In the above expression δ denotes the volume density, n the rigidity, and $\frac{m-n}{2m}$ the Poisson's ratio. Let us take $a = 10$ metres, $h = 1$ metre, $l = 30$ metres, Poisson's ratio $= \frac{1}{4}$, and therefore $m = 2n$, $\delta = 3$ and $n = 3 \times 10^{11}$ dynes per square centimetre. Then for the gravest mode ($s = 2$), $p_2 = 84 \cdot 1$ and the period $= 0 \cdot 07$ second, indicating that the cylinder will have 14 vibrations per second in the gravest mode.

If we assume a smaller length for the cylinder, say, $l = 10$ metres, then $p_2 = 108 \cdot 7$, and the period $= 0 \cdot 06$ second, showing that it will execute about 17 vibrations per second in the gravest mode. For the next mode of vibrations we get $p_3 = 272 \cdot 6$, showing that it will execute about 40 vibrations per second in this mode. The frequency of vibrations is thus seen to be of the same order as that of a solid cubical wall.

In the Tower problem, the lower end must be regarded to be under constraint, and we shall not be far out if we assume that the periods of vibrations are equal to that of a cylinder of double its length executing vibrations in the mode next to the gravest, which make the central part of the cylinder a node. We thus see that whether the building is a solid cubical structure or a hollow cylinder, the traverse vibrations are far too

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* Rayleigh, 'Theory of Sound,' vol. 1, p. 417.

quick for being confused with the microseisms. It is known that the passage of a heavily loaded vehicle will cause appreciable vibrations in a neighbouring building and that they are very quick vibrations. GUTENBERG* has, in fact, given examples and illustrations of vibrations having periods of less than a second due to commercial and industrial causes, but the vibrations which we record in the seismograms have periods varying from 10 to 30 seconds, and cannot therefore be identified with the quakes which the vibrations in buildings or trees will communicate to the ground. We must look for some other cause for the production of these quakes. We have already seen that these large period microseisms are shown in Colaba records only on days of pronounced land and sea breezes. Assuming that the activities of the meteorological factors which cause these breezes are confined to a strip of land and also a strip of sea, each 10 to 15 miles broad on either side of the coast line, they must, when they become well pronounced, cause surface waves over the sea near the coast. These waves being formed on comparatively shallow water will have periods quite different from those of the monsoon type.

Near the coast we have a sloping bed, the inclination to the horizontal being variable and in general small, except very near the coast line. The depth of water will be about 100 feet or 200 feet at a distance of 10 or 15 miles away from the coast. The analysis previously adopted for finding the effect of wind in producing surface waves over large areas of the sea and the corresponding elastic waves over the surface of the earth takes account of the finite depth of water and will therefore hold approximately for the shallow sea near the coast, provided we assume an average value for the depth between the coast line and 15 miles off the coast. The analysis, of course, makes the assumption that the sea is unlimited in extent, but in the shallow sea near the coast we have plainly a boundary. The effect of the boundary will in general be expressed by the known fact that a set of waves in advancing against a sloping beach will maintain their periods but will diminish in wave-length and speed. The periods of the waves over the earth's surface will, therefore, be given approximately, as before, by

$$p^2 = g\xi \tanh \xi d,$$

where d is the average depth of water. The observed periods of waves over the shallow sea near the coast agree fairly well with those given by this formula. For small depths, the periods of waves for different velocities of wind are easily picked up from the first three lines of Table I. The two cases of (1) an average depth of 50 feet, and (2) an average depth of 100 feet, are important, as these will represent roughly the order of the average depth of the shallow sea near the west coast. It will be seen from fig. 7 that, for the depth of 50 feet, the curve becomes asymptotic to the period-axis, when the wind velocity attains the value of 27 miles per hour, and, for the depth of 100 feet, it becomes so when the wind velocity becomes 37 miles per hour. In the first case, an

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* 'Lehrbuch der Geophysik,' p. 305.

increase of wind velocity from 20 miles per hour to 27 miles per hour will increase the period from 7 seconds to 30 seconds, and in the second case, an increase of wind velocity from 20 miles per hour to 37 miles per hour will increase the period from 6 seconds to 30 seconds. A small change in the wind velocity is thus associated with a large change in the period, and leaves ample margin for explaining the apparently wide variation in the observed period of the microseisms from 10 seconds to 30 seconds without any corresponding large variation in the wind velocity. It does not appear that the periods of 10 to 30 seconds observed in this type of microseisms can be explained on any other hypothesis.

It has already been remarked that the effect of the sloping bed will be to diminish the wave-length. Now equations (31) show that the amplitude of the microseisms varies directly as the wave-length. If, therefore, the wave-length diminishes, the amplitude will also undergo diminution. The disturbed region being so near, one should expect from Table IV that at a coastal observatory the amplitude of the microseisms should be at least 10 μ, if not 12 μ, for a wind velocity of 20 miles per hour, whereas the observed amplitude is between 1 μ and $1 \cdot 5$ μ. This large reduction in amplitude is undoubtedly due to the diminution in wave-length of the waves advancing against the sloping bed.

It is interesting to observe that, in the diurnal variation of these large period microseisms, the maximum value is reached an hour or two before the sea breeze, as recorded in the Observatory, attains the highest velocity. It is difficult to suggest an explanation. The prevalent wind, which produces these microseisms, is from the north-west, and it is probable that the maximum wind begins to affect the sea, 15 or 20 miles away from the coast, an hour or two before it arrives at the Observatory. A similar effect is also shown by the land breeze, as, indeed, is indicated by the two bottom curves of fig. 9, the reason being that the Observatory is situated at the extreme end of a narrow ridge of land projecting into the sea.

11. *Summary.*

The ground is never at rest, and a seismograph provided with an aperiodic pendulum and a large magnification will always record these ever-present movements. The types are often so complicated that it is not easy to distinguish those associated with definite weather disturbances. To obviate these difficulties, a Milne-Shaw seismograph was installed some 5 years ago in the underground constant temperature room of the Colaba Observatory, and its working condition was so arranged that it should just cease to record microseisms when the weather was undisturbed over the neighbouring seas, as in the months of January and February, when the wind velocity seldom exceeds 20 miles per hour over the sea areas. It was then noticed that microseisms made their appearance in the records whenever weather was disturbed over the Arabian Sea or the Bay of Bengal, so as to cause rough seas over a fairly wide area. In particular, three distinct types of microseisms were recognised, and these were associated with (1) the south-west monsoon, (2) the storms in the Arabian Sea and the Bay of Bengal, and (3) local disturbances, such as pronounced land and sea breezes. Those associated with the southwest monsoon are steady vibrations, having periods varying from 4 to 10 seconds, according to the strength of the air current over the sea.

To explain the periods and the amplitudes of the microseisms an expression has been obtained by mathematical analysis for the sea waves, which will be generated and maintained by a steady monsoon current, agreeing with the observed heights and periods of such waves for different wind velocities. The tremors produced and maintained at the bottom of the sea by such waves are found to be a kind of standing vibrations combining to form progressive waves, and these travel as RAYLEIGH waves on arriving overland, where the normal stress is zero. The observed periods and amplitudes of the monsoon microseisms show fairly good agreement with the theoretical values. An interesting result is obtained from purely theoretical consideration that the maximum possible value of the ratio of the amplitude of the horizontal component of microseisms to that of the vertical component at an observing station is 3, and the minimum value is 0·7, the exact value depending on its distance from the disturbed region, and this is confirmed by the observed results. The microseisms associated with storms have periods varying from 4 to 6 seconds and show typical irregular variations in amplitude owing to the superposition of waves of different periods arising on account of the existence of a marked difference in wind velocity in the storm and the surrounding areas. They make their appearance in the seismograms as soon as a storm has formed, and disappear only after it has passed inland and ceased to affect the sea.

The types are readily distinguished, and thus throw open to the meteorologists a new method of forecasting the existence of storms. The amplitudes of microseisms are found to be a function of the distance, and the intensity of the storms and curves have been given showing the nature of the relationship obtained.

During the pre-monsoon and the post-monsoon periods, when the records are almost free from monsoon microseisms, the formation and the early development of a storm are easily recognised by the gradual appearance of feeble microseisms of variable amplitude, which become more and more marked as the storm is fully developed. During the five years the instrument has been in operation several storms formed in the Arabian Sea and the Bay of Bengal, and all of them gave rise to microseisms of this kind from the time of their formation until they passed inland and ceased to disturb the sea.

The microseisms associated with a local disturbance have large periods, varying from 10 to 30 seconds, and appear to be caused by waves over the shallow sea near the coast, for such waves have periods of exactly this order. They are certainly not due to the shaking of buildings and trees by gusts of wind, for such shakings will cause vibrations which in an ordinary building will have periods less than 0·1 second.

APPENDIX A.

MICROSEISMS PRODUCED BY STORMS DURING 1926.

Serial number.	Locality of storm.	From meteorological evidence.					Microseisms.				
		Month.	Date.	Greatest observed barometric depression.	Intensity.	Course of storm and time of beginning and end of storm.	Period (average) in seconds.	Amplitude in microns (maximum)	Nature.	Duration.	Remarks.
				inches						hours	
1	Bay of Bengal	1926 May	19–23	0·70	Severe	Formed east of Ceylon and crossed coast near Akyab. Slight storm from the 19th to the 20th, severe storm on the 21st and the 22nd	5·5	1·2	Storm type (well-marked)	114	Feeble microseisms of storm type began to appear from 19 d. 14 h., gradually increased in intensity and became fairly marked from 20 d. 21 h. and attained maximum intensity on the 22nd; from the 23rd morning they began to weaken and disappeared on the morning of the 24th.
2	„	July	5–9	0·27	Slight	Formed at head of Bay on the 5th and crossed coast same date. Slight storm from the 5th to the 7th and depression from the 7th to the 9th	6·0	1·8	Storm type (well-marked)	91	Monsoon microseisms assumed storm type from 4 d. 17 h. and gradually became more and more pronounced. They became strongly pronounced from 5 d. 4 h. and remained so up to 8 d. 11 h. Afterwards they weakened and assumed monsoon type again from 8 d. 12 h.
3	„	August	13–21	0·37	Slight	Inland storm, near Ballasore, on the 16th	6·0	0·8	Mainly monsoon type	81	Microseisms mainly monsoon type except on 15–17, when they were of storm type.

APPENDIX A—(continued).

Serial number.	Locality of storm.	From meteorological evidence.					Microseisms.				
		Month.	Date.	Greatest observed barometric depression.	Intensity.	Course of storm and time of beginning and end of storm.	Period (average) in seconds.	Amplitude in microns (maximum)	Nature.	Duration.	Remarks.
4	Arabian Sea	1926 September	1–5	inches 0·25	Slight	Slight inland storm formed in Central India on September 1, moved near Deesa on the 2nd, Bhuj on the 3rd and Karachi on the 4th, became depression on the 5th, then disappeared	6·0	0·8	Storm type (feeble)	hours 85	Feeble monsoon microseisms assumed storm type from 31 d. 19 h. (August), and continued so until 4 d. 8 h. (September), after which they disappeared.
5	Bay of Bengal	,,	14–24	0·40	Slight	Formed mid-Bay and crossed coast near False Point on the 16th. Depression from the 14th to the 15th, slight storm on the 16th, inland storm from the 17th to the 24th	5·0	1·4	Storm type (well-marked)	230	Feeble storm type microseisms began to appear from 13 d. 10 h., gradually increased in intensity and became strongly pronounced from 15 d. 21 h., and remained so up to 18 d. 6 h. Thereafter they weakened and finally disappeared from 23 d. 0 h. Apparently both these storms contributed to produce the effect.
6	Arabian Sea	,,	16–21	0·32	Moderate	Formed off Kathiawar on the 16th, crossed coast near Veraval on the 17th, moved westwards to sea on the 19th and crossed coast again east of Karachi on the 21st and disappeared					
7	Bay of Bengal	October	15–18	0·45	Moderate	Formed in the Andaman Sea on the 15th and crossed coast near Kyaukpyu on the 17th	6·5	0·8	Storm type (feeble)	130	Feeble storm type microseisms began to be recorded from 14 d. 0 h. until 19 d. 10 h. after which they disappeared.

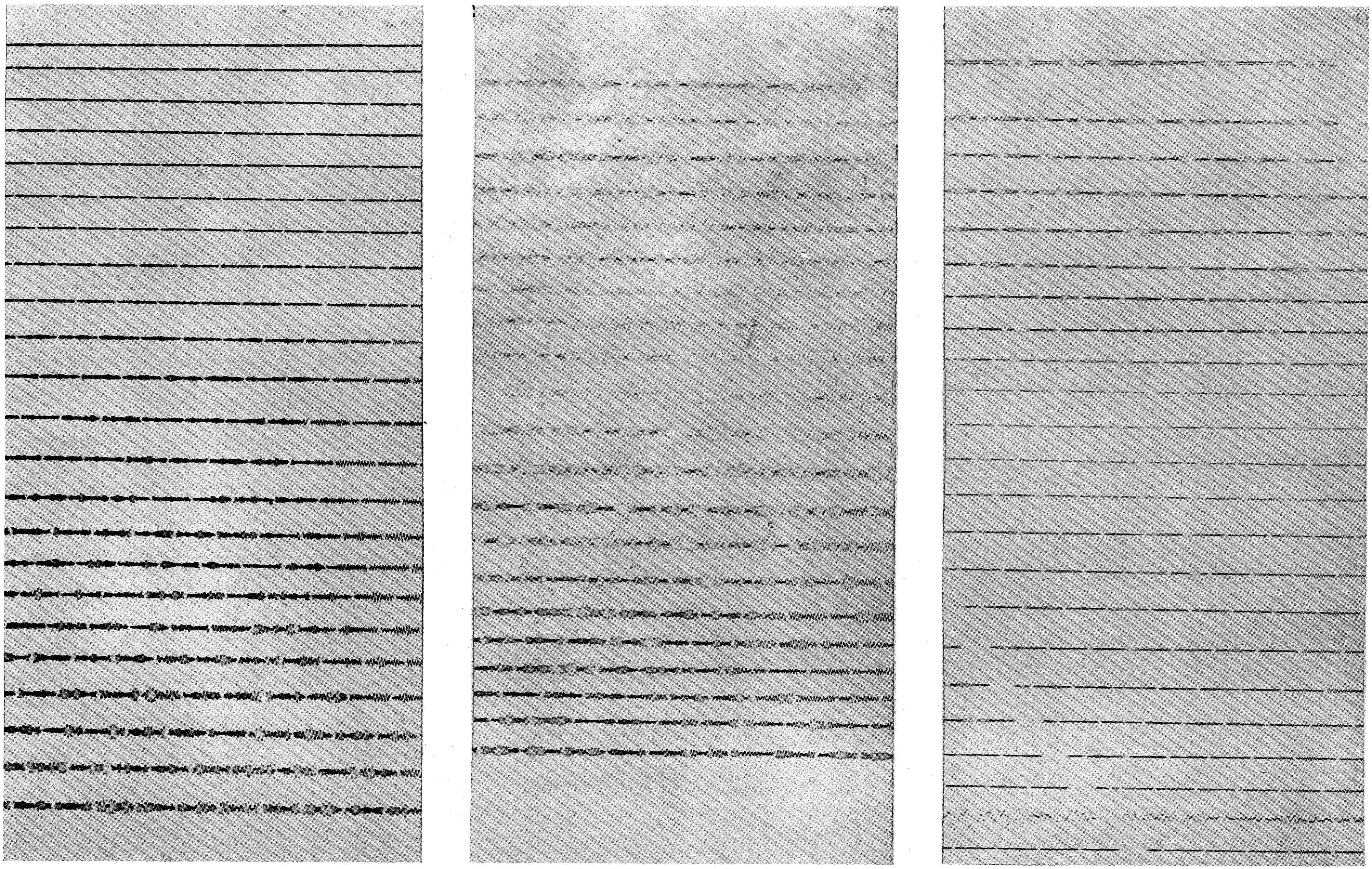
8	Arabian Sea	November	7–18	1 (about)	Severe	A depression formed over Palk Strait on 6th, and moved westwards into Arabian Sea. On 13th, SS Port Darwin (lat. 8° N, long. 55° E) reported wind of force 11 and passage of a cyclone due north of vessel. The storm weakened on approaching African coast, near 10° N, 53° E on 15th and disappeared on 17th	5·0	0·8	Storm type (feeble)	288	Feeble microseisms of storm type began to appear from the afternoon of the 5th. They continued to be recorded and remained feeble until the afternoon of the 17th, after which they disappeared.
9	Bay of Bengal	,,	21–26	0·36	Severe	Formed east of Ceylon on the 21st and crossed coast at Kyaukpyu on the 26th. Slight from the 21st to the 23rd, severe from the 23rd to the 25th and became slight again afterwards	5·5	1·4	Storm type (fairly well-marked)	162	Feeble microseisms of storm type began to appear from 20 d. 16 h., became more marked on the 22nd and continued so till the midnight of 25th. They weakened on the 26th and disappeared from 27 d. 10 h.
10	,,	December	24–28	0·30	Moderate	Formed east of Ceylon on the 24th and crossed coast near Barisal on the 27th	5·0	1·0	Storm type (fairly well-marked)	95	Feeble microseisms of storm type began to be recorded from 24 d. 17 h., became more pronounced on the 25th and continued to remain so until the afternoon of the 27th. Afterwards they weakened and disappeared on the evening of 28th.

APPENDIX B.

Relation between High Winds and Microseisms, during Winter Months, January to March and December, 1925.

Date.	Wind velocity.						Microseisms.			
	Maximum.	Duration above 20 miles per hour (average).				Direction (average).	Period (average).	Amplitude in microns (maximum).	Duration.	
		From		To					From	To
1925		h.	m.	h.	m.		second		h.	h.
January 4	28	14	0	18	20	NNW	22	2·7	10	13*
7	24	17	20	18	45	NNW	22	2·7	13	22
16	23	16	12	18	27	NNW	19	2·1	throughout the day	
17–18	28	21	43	9	45	NW	22	3·4	throughout the days	
21	25	13	30	16	0	NNW	19	2·1	10	22
22	33	11	50	21	0	NNW	18	1·9	10	22
23	24	15	0	18	55	NNW	16	2·0	9	21
February 7	27	14	27	21	0	NNW	18	2·4	9	23
11	32	11	5	20	10	NNW	20	2·9	throughout the day	
12	29	12	5	20	0	NNW	20	2·9	9	22
13	25	14	0	17	50	NNW	20	2·9	9	21
17	29	8	0	20	37	NNW	19	2·6	7	22
18	27	12	0	18	0	NNW	22	3·4	9	20
19	26	14	40	19	10	NNW	18	2·4	9	20
20	25	15	35	17	20	NNW	14	1·6	11	21
21	26	16	0	20	5	NNW	21	2·5	12	20
22	26	15	40	19	50	N	21	2·5	13	21
March 1	27	14	5	19	48	NNW	19	2·1	11	21
24	27	15	40	17	45	NNW	19	2·1	12	24
24–25	25	23	20	1	30	N	18	2·4	0	4
25	27	14	0	17	42	NNW	18	2·4	10	22
26	33	13	25	18	20	NNW	20	2·9	10	22
27	38	15	0	18	50	NNW	16	1·6	14	20
29	26	16	0	18	15	NNW	16	1·6	10	24
30	31	13	0	17	35	NNW	16	1·6	10	24
31	26	12	5	16	20	NNW	18	1·9	10	24
December 3	24	16	50	17	45	NNW	20	1·7	8	17
6	24	9	20	10	20	E	20	1·1	6	17
7	27	7	15	11	0	E	20	1·1	4	13
8	24	1	0	11	0	E	18	1·4	1	22
21	26	15	8	20	15	NNW	17	1·3	throughout the day	
22	25	16	30	19	0	NNW	21	2·5	13	22
31	25	18	37	19	45	NNW	25	3·5	11	20

* Record lost after 13 hours.

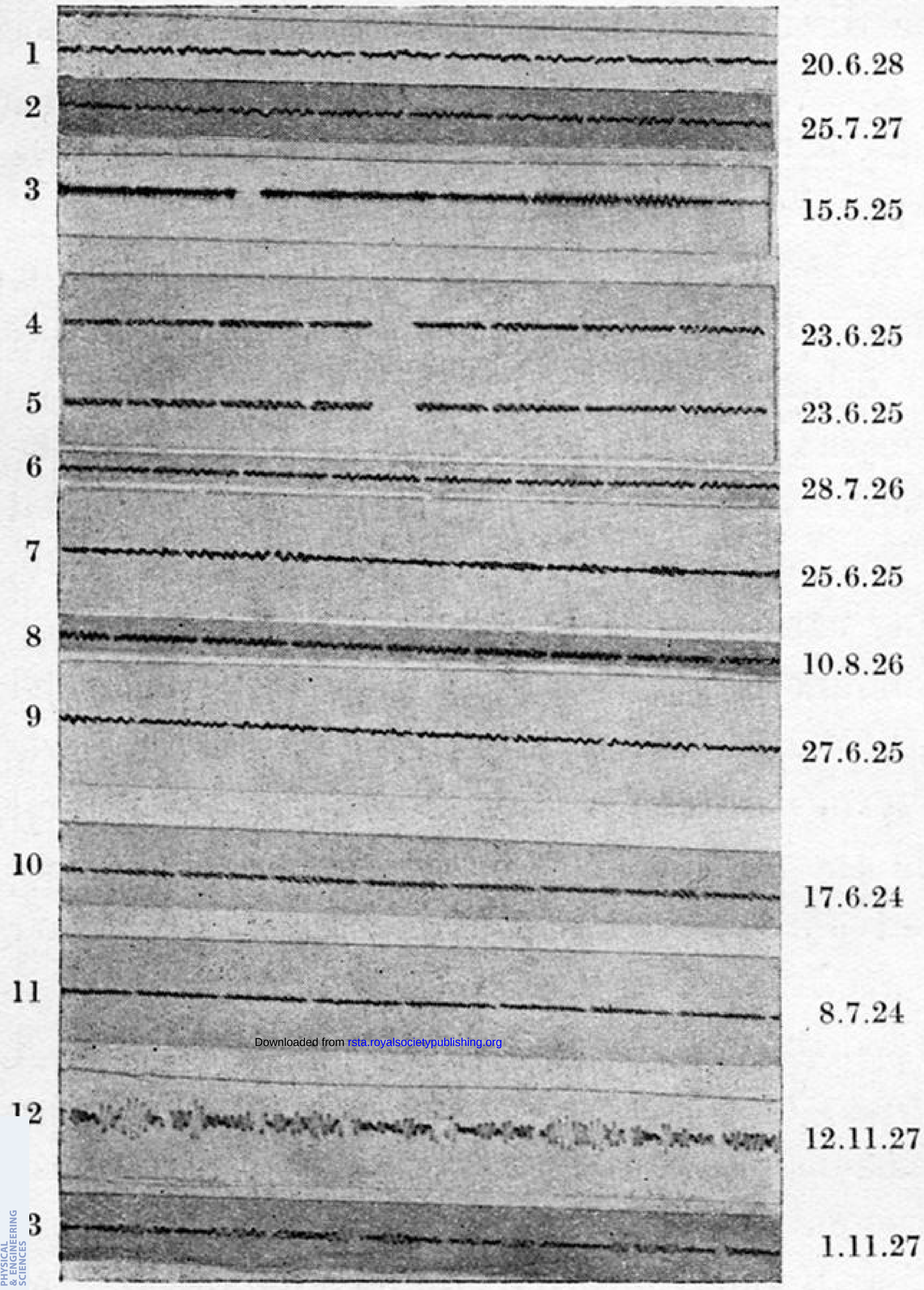


(*a*) From 12h. Nov. 11 to 10h. Nov. 12

(*b*) From 11h. Nov. 12 to 7h. Nov. 13

(*c*) From 8h. Nov. 13 to 6h. Nov. 14
(Second line from bottom shows an earthquake shock)

Records showing the incidence, development and gradual weakening of microseisms in relation to a storm, which formed in the Arabian Sea in the afternoon of 11 November, 1927, some 400 miles away from Bombay and crossed the coast between Bombay and Ratnagiri in the early hours of the 13th. The lines are cuttings for nearly 10 minutes from consecutive hourly traces. The interval between consecutive breaks in each line is equal to a minute of time.



IG. 1.—Short Period Microseisms associated with the S.W. Monsoon and Storms (N–S component). Instrument in the underground room. Interval between consecutive breaks equals one minute.

1, 2 and 4–11. Monsoon Type Microseisms.

3. Associated with a Storm in the Bay of Bengal.

4, 5, 7, 9. Associated with strong Monsoon with a depression in front moving into Kathiawar and causing rough seas off the Konkan and Kathiawar coasts.

12. Associated with a Storm in the Arabian Sea, which formed on November 11 and crossed coast between Bombay and Ratnagiri on 13th.

13. Associated with a Storm in the Bay of Bengal, which passed inland near Nellore on November 1.

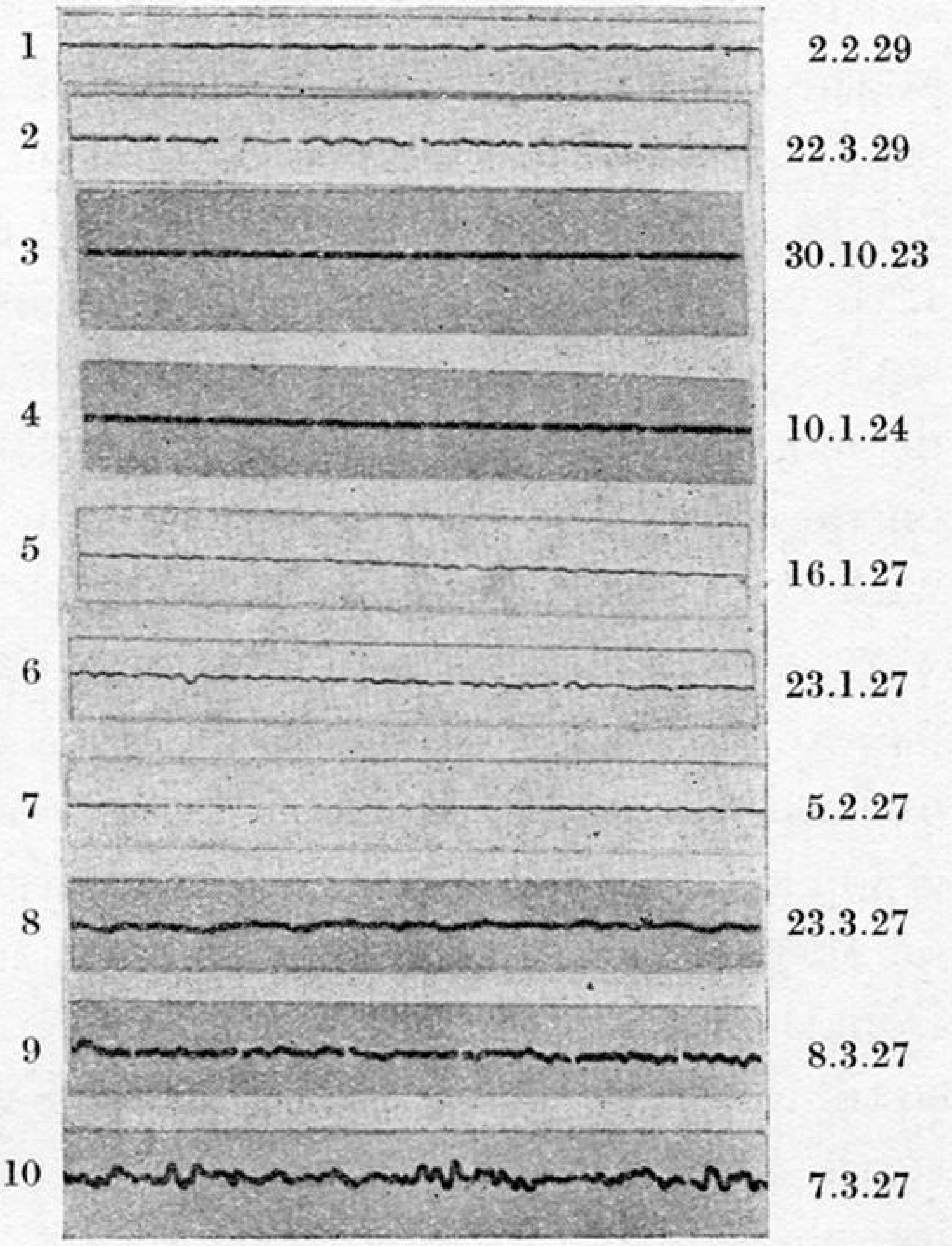


FIG. 2.—Long Period Microseisms associated with pronounced Land and Sea Breezes (N–S component). Interval between consecutive breaks equals one minute.

1, 7. Instrument in the underground room.

8–10. Instrument in an overground room.

(*a*) From 12h. Nov. 11 to 10h. Nov. 12

(*b*) From 11h. Nov. 12 to 7h. Nov. 13

(*c*) From 8h. Nov. 13 to 6h. Nov. 14
(Second line from bottom shows an earthquake shock)

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